Light

Astronomers have primary used electromagnetic radiation (i.e. light) to observe the universe. Only very recently have we found other means of observing.(i.e. gravitational waves, cosmic rays). For many many centuries light in the visible region of the spectrum was the only observable range of light. New technologies and advances in understanding have expanded the range of visible observations considerable.

Fig. 1 Which of the lights is closest? Which is the brightest? Which is the largest? It can be hard to tell.

Geometric Optics

In the absence of any interactions, light will travel in straight lines. Thus, we can use our understanding of basic geometry to interpret and predict optical phenomena.

Parallax

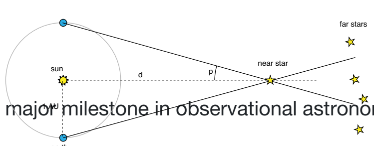


Diagram illustrating parallax due to the change in position of the earth along its orbit.

$$d = \frac{1\text{AU}}{\tan p} \simeq \frac{1}{p}\text{AU} \quad (1)$$

A major milestone in observational astronomy occured when **trigonometric parallax** was first used to measure the distance to a celestial object. Although a successful observation wouldn't occur until 1838 by Friedrich Bessel for the star 61 Cygni, astronomers had known for years that if the earth was indeed orbiting the sun, parallax should be observable. In fact, the earlier geocentric astronomers used the lack of any stellar parallax observations to support their claim that the earth was indeed motionless. With today's instruments, measuring parallax for nearby objects is easy. However, the angles are so small that these measurement would have been difficult for early astronomers.

The star measured by Bessel has a parallax angle of 285.88 ± 0.54[5] mas. (mas = milliarcseconds). This puts 61 Cygni about 11 light years away. The closest star to us, Proxima Centauri, has a parallax of 0.7687 ± 0.0003 arcsec. This is equivalent to measuring something about an inch in diameter about 4 miles away. So, from City College, measuring a quater located at the Bronx zoo would be a similar level of precision.

The parsec

We'll convert from radians to arcseconds:
1 radian = 57.2957795° = 206264.806"

Then, we'll make a new unit of distance called the **parsec**.
1 pc = 2.0624806 × 10^5 AU = 3.0856776 × 10^26 m.

$$d \simeq \frac{206,265}{p''}\mathrm{AU} \quad (2)$$

$$d = \frac{1}{p''}\mathrm{pc} \quad (3)$$

One can see from the equation that if a parallax angle is 1 arcsecond, then the distance to the star will be 1 parsec.

Light years

Light years are another unit of distance commonly used in astronomy. It's simply the distance light travels in one year in a vacuum:
1 ly = $9.460730472 \times 10^{15}$ m.
or, 1 pc = 3.26 ly

Cosmic distances

Object	meters	AU	Light Year	Parsec
Sun	1.496×10^{11}	1	1.58×10^{-5}	4.85×10^{-6}
Pluto	7.5×10^{12}	50.1	-	-
Voyager 1	2.089×10^{13}	139.6	-	-
Proxima Centuri	4.014×10^{16}	2.69×10^{5}	4.2	1.301
Center of Milky Way	2.3×10^{20}	-	25,000	7,600
Andromeda Galaxy	2.4×10^{22}	-	2.5×10^{6}	760000

The farthest we can measure using parallax is about 1 kpc, or .001". This means we can only sample *nearby* stars using this measurement tool. There was a planned NASA mission called the Space Interferometry Mission that was designed to measure 4 μarcseconds (.000004") which would have yielded parallax measurements as far as 250 kpc away. However, this mission was canceled. Fortunately, the GAIA spacecraft built by the European Space Agency was launched in 2013 and is now releasing data. It can measure about 7 μarcseconds for some of the brightest stars. It will eventually catalog about 1 billion stars from the milky way (about 1 % of the total number of stars in the galaxy).

Magnitude Scale

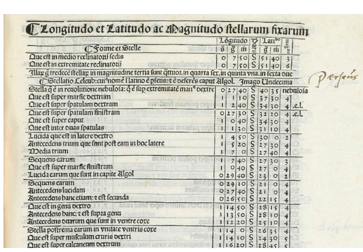


Fig. 2 Ancient Magnitue Scale: 1 brightest -> 6 dimmest. This catalog is from the 1515 edition of Ptolemy's Almagest.

One of the first things the earliest astronomers probably noticed was that some stars are brighter than other stars. As ancient science was mostly about categorizing things, the greek astronomer Hipparchus assigned stars to different categories based on their brightness. The oldest catalog of star magnitudes we have today is from Ptolemy's Almagest.

Apparent Magnitude

m indicates how bright it looks to us. For the ancients, $m = 1$ was the brightest star (not the sun). $m = 6$ was the dimmest visible star.

Today, we have a much more fine grained magnitude scale. The sun is very bright, and has a value of -26.83. The faintest objects are approximately m = 30.

Every 5 units in the magnitude scale corresponds to 100 times a change in brightness.

Describing Brightness

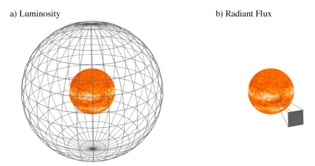


Fig. 3 a) Luminosity is a measure of the entire output of the star. b) radiant flux is what we see as the brightness, since it depends on how far away the observer is.

Radiant Flux, F is what we mean when we say brightness. Total amount of energy at all wavelengths that crosses a unit area perpendicular to the direction of the light's travel per unit time. Or, energy per second received from a star by 1 square meter. or Watts/Meter^2.

Luminosity, L, energy emitted per second. This is intrinsic to the star and doesn't depend on the observer's position.

$$F = \frac{L}{4\pi r^2} \quad (4)$$

can be used to find the flux F at a distance r away from a source with luminosity L.

This is an inverse square law.

Example Problem #1:

The radiant flux above that reaches the earth from the sun was measured to be on average about 1361 W/m^2 [ref] (This is above the atmosphere). What is the luminosity of the sun based on this measurement?

Since $L = 4\pi r^2 F$ where r is the Earth-Sun distance: $1\ \mathrm{AU} = 1.496 \times 10^{11}\ \mathrm{m}$, we obtain $3.83 \times 10^{26}\ \mathrm{W}$

Absolute Magnitude

If we put every star at a distance of 10 pcs away from us, then ranked their magnitudes, we could have a scale that gave an **absolute magnitude** (M).

Since a star that is 100 times brighter will have an apparent magnitude difference of 5, we can write as a ratio between the flux from two stars:

$$\frac{F_2}{F_1} = 100^{(m_1 - m_2/5)} \quad (5)$$

or, if we take the log of both sides:

$$m_1 - m_2 = -2.5 \log_{10}\left(\frac{F_1}{F_2}\right) \quad (6)$$

The we combine eqs. (4) and (5) to obtain:

$$100^{(m-M)/5} = \frac{F_{10}}{F} = \left(\frac{d}{10\ \mathrm{pc}}\right)^2$$

Solving for d

$$d = 10^{(m-M+5)/5}\mathrm{pc} \quad (7)$$

$m - M$ is effectively a measure of the distance to the star and is called the **distance modulus**:

$$m - M = 5\log_{10}(d) - 5 = 5\log_{10}\left(\frac{d}{10\ \mathrm{pc}}\right) \quad (8)$$

Example Problem

#2:

What is the absolute magnitude of the sun?

If we rearrange eq. (8)

$$M_{\text{Sun}} = m_{\text{Sun}} - 5\log_{10}(d) + 5 \quad (9)$$

From the tables of information: ($m_{\text{Sun}} = -26.83; d = 1\ \text{AU} = 4.848 \times 10^{-6}\ \text{pc}$) Thus $M_{\text{Sun}} = +4.74$

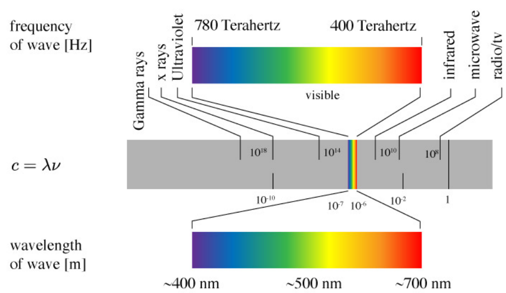
For other bodies we'll obtain:

$$M = M_{\text{Sun}} - 2.5\log_{10}\left(\frac{L}{L_\odot}\right) \quad (10)$$

Light as a Wave

$$c = \lambda\nu \quad (11)$$

It was becoming apparent around the time of Newton that the notion of light as a little particle that moved across rooms like bullets was not quite adequate to explain all the observed phenomena. There was some disagreement between physicists about light, but no one could really provide any unshakeable evidence to confirm that light was either a particle or a wave.



Region	Min λ	Max λ
Gamma Rays	-	1 nm
X-Ray	1 nm	10 nm
Ultraviolet	10 nm	400 nm
Visible	400 nm	700 nm
Infrared	700 nm	1 mm
Microwave	1 mm	10 cm
Radio	10 cm	-

Double Slit

The double slit experiment that was done by Thomas Young in 1801 was the first conclusive proof that light exhibited properties only explainable if it was a wave.

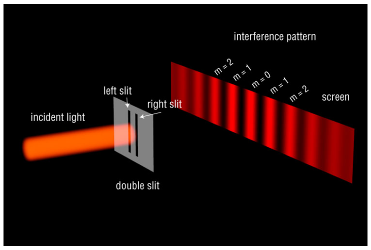


Fig. 4 The classic double slit setup

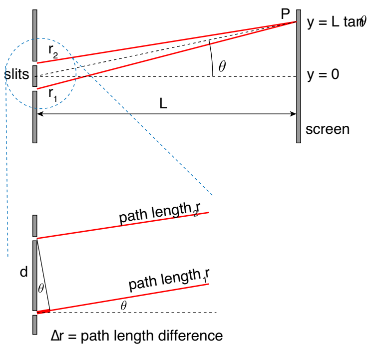


Fig. 5 The geometry of the double slit experiment.

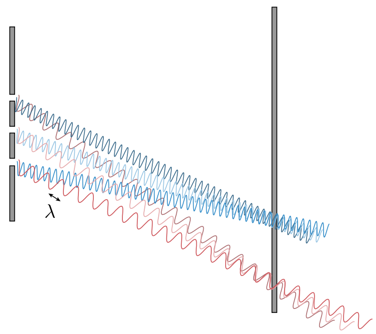


Fig. 6 Drawing showing how blue light will have a first maximum closer to the optical axis.

The interference of light allows us to use diffraction gratings to separate light by wavelength.

Fig. 7 Electromagnetic Radiation is explained by oscillating electric and magnetic fields

The Poynting Vector

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} \quad (12)$$

The rate at which energy is carried by a light wave is called the **Poynting Vector** (after John Henry Poynting (1852-1914)). It points in the direction of the EM wave's propagation. The magnitude describes the amount of energy per unity time that crosses a unit area. More often, we would be interested in the *average* magnitude of the Poynting Vector:

$$\langle S\rangle = \frac{1}{2\mu_0}E_0 B_0 \quad (13)$$

E_0 and B_0 are the max values of the electric and magnetic fields. The time-averaged Poynting vector tells us about the radiant flux at particular frequency. Since an astronomical source might emit light in many frequencies, we need to be clear what ν we are talking about with the Poynting Vector.

Radiation Pressure

Absorption

$$F_{\text{rad}} = \frac{\langle S\rangle A}{c}\cos\theta \quad (14)$$

Reflection

$$F_{\text{rad}} = \frac{2\langle S\rangle A}{c}\cos^2\theta \quad (15)$$

Light exerts a force: **Radiation Pressure** In general, the force due to the radiation pressure is not very strong. However, it can add up over time.

Example Problem #3:

Design a solar sail based propulsion system. The spacecraft has a mass of 100 kg and we would like an acceleration of 5 m/s^2.

Using the second law: $F = ma$. If we have a 100 kg spacecraft and we want an acceleration of 5 m/s^2, then our force needs to be 500 N. Setting this equal to the desired radiation pressure: (using eq. (14))

$$500 \text{ N} = F_{\text{rad}} = \frac{\langle S\rangle A}{c}\cos\theta \quad (16)$$

We can assume the angle between the Poynting vector and the sail's surface normal is 0 degrees. The $\langle S\rangle$ is just the average energy/time per square meter delivered by the sun:

$$\langle S\rangle = 1365 \text{ W/ m}^2 \quad (17)$$

and A is the area of the sail, which assuming a circular sail would be given by:$a = \pi r^2$. Putting all this together and solving for r:

$$r = \sqrt{\frac{cma}{\pi\langle S\rangle}} = 5.9 \text{ km} \quad (18)$$

That's probably too large...

Solar Sail Examples

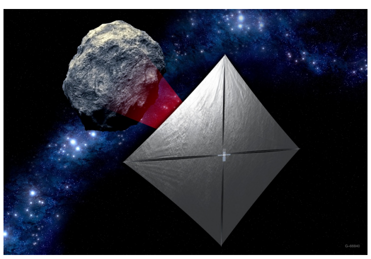


Fig. 8 Near-Earth Asteroid Scout (NEA Scout), mission concept. Expected to launch in Dec 2019

Here is the concept image for a NASA mission that will attach a solar sail to a CUBE-SAT for investigating near earth asteroids.

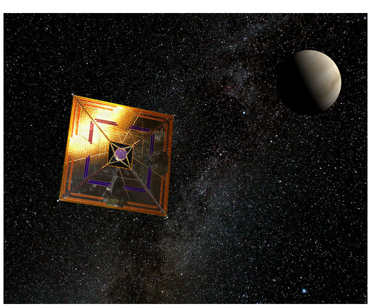


Fig. 9 The IKAROS mission.

By Andrzej Mirecki - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=14656159

The Japanese IKAROS mission

Blackbody Radiation

A **blackbody** is an *ideal emitter* that reflects no light (i.e. black). It emits energy with the characteristic spectrum shown below.

$$\lambda_{\text{max}} T = 0.0028977729 \text{ m} \cdot \text{K} \quad (19)$$

A blackbody emits what is called a **continuous spectrum**, meaning it emits some energy at every wavelength. For any given temperature, there will be a peak wavelength given by **Wien's displacement law:**

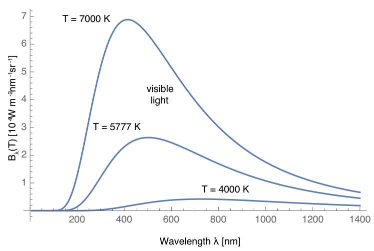


Fig. 10 The spectrum of a blackbody emitter following the Planck function.

Orion Constellation

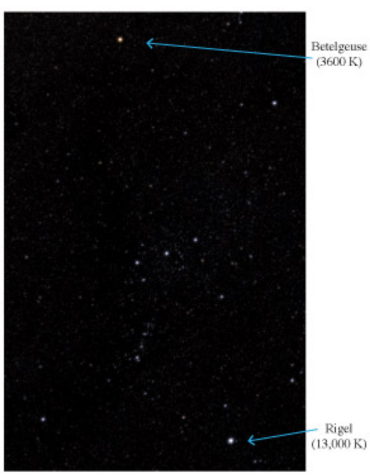


Fig. 11 The constellation Orion.

(From the Tycho 2 star map ref)

One can usually see some color difference in the stars that make up the constellation Orion. This tells us that they have different temperatures.

Stefan-Boltzmann equation

$$L = A\sigma T^4 \quad (20)$$

or for a spherical star of radius R:

$$L = 4\pi R^2 \sigma T_e^4 \quad (21)$$

The **Stefan-Boltzmann equation** relates luminosity and temperature.

Example Problem #4:

Find the peak of the sun's spectrum using its Luminosity and Radius.

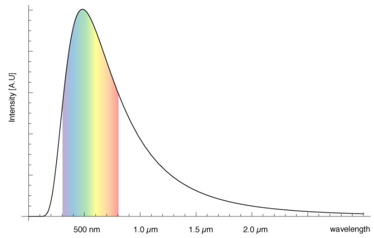


Fig. 12

We found the luminosity earlier to be: $L_\odot = 3.839 \times 10^{26}$ W. We can use the Stefan-Boltzmann equation (21): $L = 4\pi R^2 \sigma T_e^4$ to obtain the $T_\odot$:

$$T_\odot = \left(\frac{L_\odot}{4\pi R_\odot^2 \sigma}\right)^{\frac{1}{4}} = 5777 \text{ K} \quad (22)$$

Thus from Wien's displacement law:

$$\lambda_{\text{max}} \approx \frac{0.0029 \text{ m K}}{5777 \text{ K}} = 501.6 \text{ nm} \quad (23)$$

Example Problem #5:

What's the blackbody radiation of a person? What color light do people emit? (i.e. find the peak wavelength for an average human.)

An average person might be considered a 1.77 meter tall cylinder with a radius of .15 meters. This leads to surface area of about 1.9 m^2. First we'll calculate the luminosity based on this and a skin temperature of 92 F (306 K).

$$L = A\sigma T^4 = 1.93 \text{ m}^2 \times 5.670 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \times (307 \text{ K})^4 = 959 \text{ W} \quad (24)$$

Next, we can use Wien's displacement law (19) above to get the wavelength:

$$\lambda_{\text{max}} = \frac{0.0028978 \text{ m K}}{306 \text{ K}} = 9.439 \times 10^{-6} = 9,439 \text{ nm} \quad (25)$$

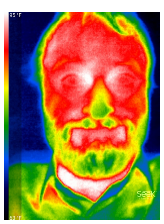


Fig. 13 A thermal image of a physicist.

Quantization

Rayleigh-Jeans

$$B_\lambda(T) \simeq \frac{2ckT}{\lambda^4}$$

Wien's approximation

$$B_\lambda(T) \simeq \frac{2hc^2}{\lambda^5} e^{\frac{-hc}{\lambda kT}}$$

All was not well in physics around 120 years ago.

There were two functions that physicists had to describe blackbody radiation. One known as Rayleigh-Jeans

$$B_\lambda(T) \simeq \frac{2ckT}{\lambda^4} \quad (26)$$

and the other called Wien's approximation

$$B_\lambda(T) \simeq \frac{2hc^2}{\lambda^5} e^{\frac{-hc}{\lambda kT}} \quad (27)$$

The Rayleigh-Jeans law was not perfect because as the wavelength decreases towards zero, this relation shows that the radiance energy would increase to infinity - obviously problematic.

Wien's Approximation was unable to predict correctly the energies at large wavelengths.

And so there were two different relations, one for short wavelengths, and one that worked for long wavelengths. This is never desirable for physicists. One equation that works for *all* wavelengths is better!

Experimentally, it was known that the following limiting conditions must apply.

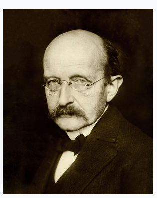
1. At a fixed temperature, the radiance approaches zero as the wavelength goes to zero.
2. At a fixed temperature, the radiance approaches zero as the wavelength goes to infinity.
3. At a fixed wavelength, the radiance approaches zero as the temperature goes to zero.
4. At a fixed wavelength, the radiance approaches infinity as the temperature goes to infinity.

Wien's approximation satisfies the first three but not the last. Rayleigh-Jeans satisfies the last three, but not the first.

Planck function

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1} \quad (28)$$

Max Planck



Max Planck was a German Theoretical Physicist. Among his many contributions, was the first showing that energy should come in discrete quanta. This turned out to be the origins of quantum theory. He was however, somewhat conservative about his own discoveries and was reluctant to fully embrace the ideas of quantum mechanics, even though he trusted his own work enough to know it must be true.

Fig. 14 Max Planck [1858-1947]

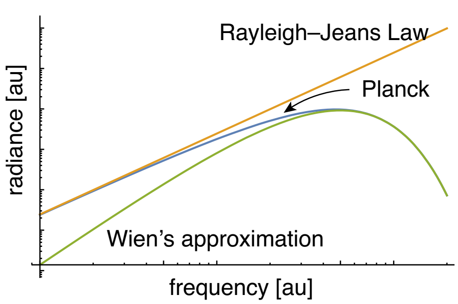


Fig. 15 Comparing the three functions for a radiator with temperature of 0.008 Kelvin. Wien's approximation fails in the low frequency limit, while Rayleigh-Jeans fails in the high-frequency. Planck's function fits both!

Example Problem #6:

Show that the maximum of the Planck function gives Wien's displacement

The Planck function is:

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

We need to take the derivative w.r.t. λ:

$$\frac{dB_\lambda}{d\lambda} = (-5)\left(\frac{2hc^2}{\lambda^6}\right)\frac{1}{e^{\frac{hc}{k\lambda T}} - 1} + \frac{2hc^2}{\lambda^5}\frac{-1}{\left(e^{\frac{hc}{k\lambda T}} - 1\right)^2} \cdot e^{\frac{hc}{k\lambda T}}\left(\frac{-1}{\lambda^2}\right)\left(\frac{hc}{kT}\right) \quad (29)$$

and then set it equal to 0.

$$\frac{dB_\lambda}{d\lambda} = 0 \quad (30)$$

Doing this and simplifying:

$$-5 + \frac{1}{e^{\frac{hc}{k\lambda T}} - 1} e^{\frac{hc}{k\lambda T}}\left(\frac{hc}{k\lambda T}\right) = 0 \quad (31)$$

Let's set

$$\frac{hc}{k\lambda T} = x \quad (32)$$

Then we get for the above:

$$5\left(e^x - 1\right) = xe^x \quad (33)$$

which is not solvable using algebraic means. We must use computers!

Some mathematica code to do a numerical solution:

```
NSolve[5 (Exp[x] - 1) == x Exp[x], x]
```

yields $x = 4.965$ Thus, returning to the constants:

$$\frac{hc}{k\lambda T} = 4.965 \quad (34)$$

or

$$\lambda T = \frac{hc}{4.965\ k} \quad (35)$$

Plugging in the values for h, c, and k gives us:

$$\lambda T = 0.002897\ \mathrm{m} \cdot \mathrm{K} \quad (36)$$