CCNY PHYS 35100 - Fall 2024 - Quiz 1 Name Solutions

Sep 16, 2024

Instructions: There are 3 questions. Please show your work with as much detail as you can. Each part is worth the same amount. No calculators or notes.

1. Tortured Physicists Department

Work out the first 3 terms for the Taylor series expansions of these two functions, around the point x = 0. (Terms that are =0 don't count in the tally) You don't need to write the summation series representation, just the polynomial.

a. $f(x) = \ln(1+x)$

b. $f(x) = e^{x^2}$

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''}{2!}(a)(x-a)^2 + \frac{f'''}{3!}(a)(x-a)^3 + \frac{f''''(a)}{4!}(x-a)^4$$

a) around point $x=0$

$f(0) = \ln(1) = 0$

$f'(x) = \frac{1}{1+x} \rightarrow \frac{1}{1}$

$f''(x) = \frac{-1}{(1+x)^2}$

$f'''(x) = \frac{2}{(1+x)^3}$

$$\boxed{\ln(1+x) \approx 0 + 1x - \frac{x^2}{2} + \frac{x^3}{3}}$$

b) take derivatives

$f(0) = 1$

$f'(x) = 2xe^{x^2} \overset{x=0}{\Rightarrow} 0$

$f''(x) = 2e^{x^2} + 4x^2e^{x^2}$

$= 2e^{x^2}(1+2x^2) \overset{x=0}{\Rightarrow} 2$

$f'''(x) = 4xe^{x^2}(1+2x^2) + 2e^{x^2}(4x) \overset{x=0}{\Rightarrow} 0$

$f''''(x) = (4e^{x^2} + 4xe^{x^2})(1+2x^2) + 4xe^{x^2}(4x) + 4xe^{x^2}(4x) + 2e^{x^2}(4)$

$\overset{x=0}{=} 4 + 2 \times 4 = 12$

most terms are zero

$e^{x^2} \approx 1 + 0 + \frac{2}{2}x^2 + 0 + \frac{12}{4!}x^4$

$$\boxed{\approx 1 + x^2 + \frac{x^4}{2}}$$

2. Look What You Made Me Do

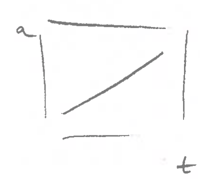
The top plot shows the position of an object being controlled by some force that changes linearly as a function of time. On the axes below, sketch qualitative plots of:

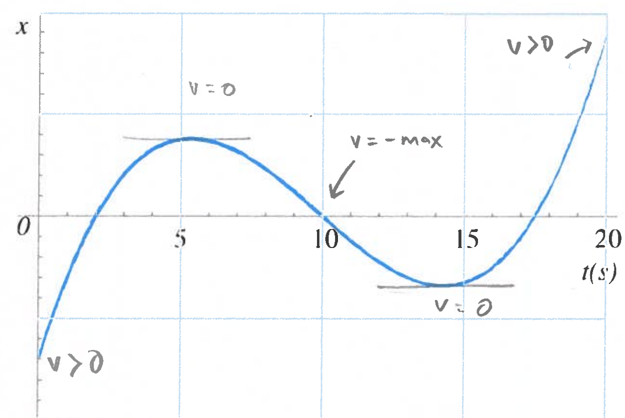
a. The object's velocity

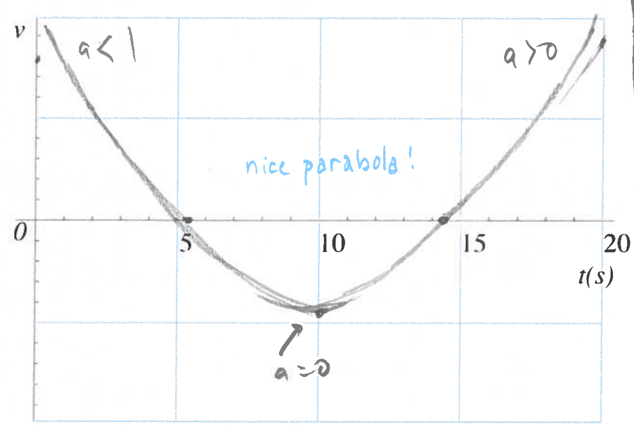
b. The object's acceleration

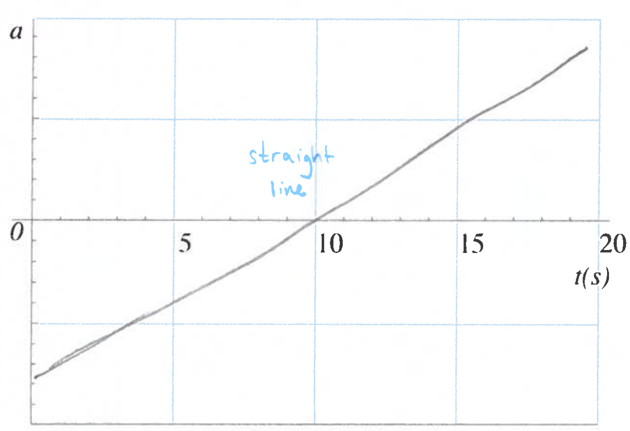
(Qualitative in this case means that it should convey the velocity and acceleration by having the right shape given the temporal horizontal axis, go through 0 when the values are 0, but the exact calibration of the vertical units doesn't matter)

$F \propto t$, therefore, a must also be linear (ie, straight line)









3. The Archer

This problem considers an arrow launched vertically upward from the ground. All motion takes place relatively close to the surface of the Earth.

a. The arrow is first launched upward with speed v_0 with no drag forces acting, i.e. in vacuum. Write out Newton's second law and solve the equation of motion (using differential equation notations like we've done) to obtain an expression for the time it takes to reach the top of the trajectory.

$$ma = F = -mg$$

$$\therefore \frac{dv}{dt} = -g$$

$$\int_{v_0}^{0} dv = -g \int_{0}^{t} dt$$

$$-v_0 = -gt$$

$$\boxed{\therefore t = \frac{v_0}{g}}$$

b. Now, include a drag force that is proportional to the velocity: $f_d = -bv$, where b is a positive constant. Repeat the process above to obtain an expression for the time it takes to reach the top of the trajectory.

$$ma = F = -mg - bv$$

$$\therefore \frac{dv}{dt} = -g - \frac{b}{m}v$$

$$dv = -g\left(1 + \frac{bv}{mg}\right) dt$$

$$\int_{v_0}^{0} \frac{dv}{1 + \frac{bv}{mg}} = -g \int_{0}^{t} dt$$

$$\ln\left(1 + \frac{bv}{mg}\right) \frac{gm}{b} \Big]_{v_0}^{0} = \left(\ln(1) - \ln\left(1 + \frac{bv_0}{mg}\right)\right) \frac{gm}{b}$$

$$\rightarrow -\ln\left(1 + \frac{bv_0}{mg}\right) \frac{gm}{b} = -gt$$

$$\therefore t = \frac{m}{b}\left(\ln\left(1 + \frac{bv_0}{mg}\right)\right)$$

c. Show that in the case of negligible drag, your result from part b) reproduces the in vacuo case from part a). (Results from Q1 might be useful)

$$t = \underbrace{\frac{m}{b}\left(\ln\left(1 + \frac{bv}{mg}\right)\right)}$$

if $b \ll 1$, expand this using taylor (#1a)

$$\ln\left(1 + \frac{bv_0}{mg}\right) \approx \frac{bv_0}{mg} + \cancel{\frac{1}{2}\left(\frac{bv}{mg}\right)^2}$$ ignore

$$\therefore t \approx \frac{m}{b}\left(\frac{bv}{mg}\right) \approx \frac{v_0}{g}$$, which is the same as in part a)