Origins of Classical Mechanics

1. Preliminaries

1.1 1d Equation of Motion for Constant Acceleration

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \tag{1}$$

Where did this come from?

You likely spent a fair amount of time working with this equation in your introductory mechanics class. Hopefully you derived it from the basic definitions of acceleration and velocity and didn't just pick it from a list of possible equations when you needed to solve a kinematics problem. In the context of this class, we'll see it as one possible solution to a more general differential equation:

$$\frac{d^2x}{dt^2} = \frac{F}{m} \tag{2}$$

where F and m are both constant in time. There will be many situations where they are not. Also, you should appreciate that it is just a 2nd order polynomial with an independent variable t, or, put another way, a parabola in which the coefficients a, v_0, and x_0 filling the normal A, B, and C values. Sometimes these more general notions are lost during introductory treatments of mechanics.

1.2 What does "Classical" mean?

We often use the adjectives *classical* and *quantum* to describe two different regimes of physics study. What's the difference?

How much do we know?

Where is this ball located right now? Be precise as possible.

What is its velocity? Be precise as possible.

"We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes."

Pierre Simon, Marquis de Laplace. (~1814)

The quote above is often referred to as "Laplace's Demon". It summarizes the idea of a **deterministic** universe, where, if we could only measure things precisely enough, and had unlimited computational power, we could predict the entirety of the universe's future. From his work: A Philosophical Essay on Probabilities

Example Problem #1:

A particle is moving in the x-direction and is confined between two perfectly bouncy (ahem, elastic) walls so that it bounces back and forth between $x = 0$ and $x = l$, where l is the distance between the two walls. If there is an uncertainty in the initial velocity measurement, i.e. Δv_0, find the subsequent uncertainty in position at time t later.

The position of an object is given by:

$$x = v_0 t$$

If the velocity has an uncertainty in the initial measurement, then there is an uncertainty in position given by

$$\Delta x = (v_0 + \Delta v_0)\, t - v_0 t$$

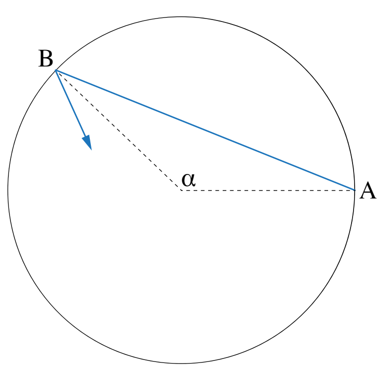
which indicates that

$$\Delta x = \Delta v_0 \; t$$

This suggests that after a time $t_c = \frac{l}{\Delta v_0}$, whatever those numbers might be, the uncertainty of the particle will be equal to l , meaning its position is completely undetermined.

Example Problem #2:

Let a particle start from point A and move towards B. Assuming this is a perfect circle and that collisions with the walls are totally elastic, will it ever come back to point A? If so, how many bounces will it take?



A particle bounces off the inner walls of a circular constraint.

Consider the angle α as the some part of the whole circle, 2π. If it's one quarter of the circle (i.e. 90°), then it would be

$$\alpha = 2\pi \left(\frac{1}{4}\right)$$

thus:

$$4 \times \alpha = 2\pi \times 1$$

After 4 reflections, it will have gone around once. We can abstract this a bit and say that:

$$\alpha = 2\pi \frac{p}{q}$$

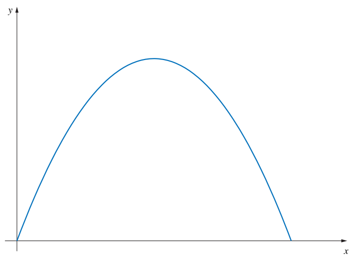
where $\frac{p}{q}$ is any rational number.

But what if $\alpha = 2\pi \times$ an irrational number?

Then, particle A will never return to the exact starting point. Interesting. One system is closed and periodic, the other is open and non-periodic. All just by changing the initial velocity.

1.3 Time

If we know how things move in the forward direction of time, can we also know how the process would happen in reverse?



2. Review

2.1 The Outline

1. Newtonian Mechanics
2. Special Relativity
3. Variational Principle
4. Lagrangian & Hamiltonian Mechanics
5. Gravitation
6. Accelerating Reference Frames
7. Rigid Body Dynamics
8. Coupled Oscillators
9. ...and ?

2.2 Tools

- Everything from 20700 and 20800
- Advanced Calculus and other Math
- Computers

Intro Physics Review

Largely Mechanics (i.e. 20700) but with some references to electric fields and other 20800 material.

Example Problem #3:

What are Newton's 3 Laws of motion?

Example Problem #4:

What are the implications of the conservation of Energy?

... and Momentum?

... and Angular Momentum?

Example Problem #5:

What goes on the R.H.S.?

> $$\frac{dU}{dx} = ?$$

Math you'll need

Differential Equations

$$\frac{dv}{dt} = \frac{F}{m}$$

But what if F is messy?

This is Newton's second law. It says the derivative of velocity w.r.t time is equal to the net force acting on the object divided by the mass. In intro physics, we mostly never had anything funny on the R.H.S. of the equation. It was generally a constant F, or perhaps something like a single sine or cosine with a t in the argument. Now, we're going to have all sorts of things in that F. They might be time dependant, velocity dependant, etc. That makes solving the equation more involved in many cases, and just plain impossible in others. Well, impossible for a analytical approach, but we can brute force it with a computational method.

Taylor Series

The Taylor series is generally defined as: $f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$

For example, for the exponential function:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Vectors

We'll need more complicated vector math to deal with different coordinate systems.

To open the door, which direction should the torque vector point?

Linear Algebra

Later, we will express some of the physics in the context of Linear Algebra.

Computational Physics

Quick, you have 5 minutes to prepare a plot of:

$$U(x) = -\frac{1}{x}e^{-x} + \frac{1}{5x^2}$$

What do you do?

3. Evolution into Modern Times

Just calling this topic 'classical' suggests it is old. Parts of it indeed are quite old. Some parts are not. But, just because something is old doesn't mean its irrelevant or not worth our time as modern practitioners of physics. Much of modern physics relies on the ideas, methods, and frameworks of classical mechanics. And all of classical mechanics is still correct*, within certain limits.

* In a philosophy of science class, one might spend a lot time discussing what it means to be *correct*. Is there absolute truth? A topic for another day.

3.1 Tools evolve

Analytical vs. Numerical

> **Analytic Solutions**
>
> Sometimes we can obtain an exact solution that describes the motion.

> **Numeric Solutions**
>
> Many more situations will not have an exact, algebraic solution and will require other methods, i.e. numerical methods.

Both of these approaches are useful for physics. When algebraic solutions are obtainable, then it's often easier to understand the system in more fundamental ways. One can see how it will evolve, what the various influences on the system are (and aren't), and the generally satisfy our desire for some degree of 'beauty' in the math. Nature is reduced to simple equations in these contexts. However, at this point, most of the situations that lead to clean algebraic results have already been discovered and investigated so there's not much *new* to do there.

Many more systems will not produce exact algebraic solutions though. These need to be approached through other means. Even some very basic sounding situations are not solvable through algebraic means. For example, just asking when a certain planet in an elliptical orbit will be at a certain place, can lead to a transcendental equation that has to be solved algorithmically or through other approximation methods. Or, as well see soon, figuring out how far a projectile will travel with linear air resistance cannot be done without numerical methods.

Thus for this course, we will work with both approaches simultaneously. We will study many classic cases of dynamics that yield exact solutions, and at the same time, learn how to approach them in other ways that will help tackle the other, less well behaved systems.