I. Numerical Methods - A very brief introduction

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 - 1. Analytic vs. Numeric
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I.I Analytic vs. Numeric

$$rac{dv}{dt} = a = rac{F}{m}$$

In the case of free fall, this becomes:

$$\dot{v} = g$$

The above equation, $\dot{v} = g$ is technically a differential equation, it just happens to be a very easy one to solve.

$$rac{dv}{dt} = g$$

Then doing Separation of Variables

$$dv = gdt$$

Integrating both sides:

$$\int_{v_O}^v dv = \int_0^t g \; dt$$

leads to the familiar equation for velocity of a falling object:

$$v(t) = v_0 + gt$$

This is the analytical solution for velocity using Netwon's second law for the case of a constant acceleration.

The position of the object would can also be solved analytically and you would obtain the familiar 2nd order polynomial:

$$x(t)=x_0+v_0t+\frac{1}{2}gt^2$$

More Analytic Solutions:

If F = -bv, then we can solve for velocity and obtain:

$$v(t) = v_0 e^{rac{t}{ au}}$$

This is another analytic solution, to a slightly more complicated ΣF situation. The harmonic oscillator is another example of an analytic solution:

$$v(t) = A\omega\sin(\omega t + \phi)$$

You've seen a bunch already in fact.

But, what about when

$$\int dv = \int {
m (Something Nasty)} \, dt$$

There are many cases in all the branches of physics where the physical system does not lead to a analytically solvable equation. It doesn't mean there's anything wrong with physics, or math. It's perfectly ok. It just means we can't express the answer in a nice way.

I.2 Numerical Methods

Re-writing our original definition of acceleration, we can say that:

$$rac{\Delta v}{\Delta t} = a$$

This is the discrete form, where the differentials are expressed as deltas instead. Expanding out the deltas we would get:

$$v_f - v_i = a(t_f - t_i) = a \Delta t$$

This is essentially how you did physics in HS or at the pre-calc level. But, it can be used to do much more advanced physics too.

Let's say we know a value for v_i

$$v_f - v_i = a\Delta t$$

Then we can just solve this to get a value for v_f at some Δt .







It will help to consider what we are actually doing when we make that calculation. We are really just finding the area under the a(t) curve. That's what integration is. So the velocity is just the geometric area under the acceleration curve. All we have to do is calculate the area.

The Area under the curve

Numerical Integration

Let's start with $v_i = 0$. Then we can see that

$$v_f = a \Delta t$$

That the area of the first rectangle in the previous graph. Now we can use that v_f in the original equation as the v_i , and we will obtain:

$$v_f = v_i + a\Delta t = a\Delta t + a\Delta t$$

... and so on.

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Likewise, we can do the exact same thing for $\frac{dx}{dt} = v$. We calculate the area under the velocity curve to obtain the position at every Δt .

We can see right away though, that our area is a little trickier to calculate here. Now we have triangles!

But, we know the area of a triangle so everything is manageable. We just need to be careful with our choice of velocity.