# PHYS 35100 - FALL 2024

# Homework Set 3

Due Oct 30 via Blackboard submission, before 12:00 pm.

#### 1. t is short, but how short. [A]

In the Brachistochrone Problem, we found the path of least time for an object sliding from a high point to a lower point to be a cycloid. Now, prove another interesting feature for object sliding down this curve: the time to reach the bottom is independent of the location on the cycloid curve.

To do this, start with the following parametric equations for the cycloid:

$$x = \frac{a}{2}(\theta - \sin\theta) \text{ and } y = \frac{a}{2}(1 - \cos\theta) \tag{1.1}$$

a. Show that the time to reach the bottom from an initial point  $B_0$  is given by the integral:

$$t(B_0 \to B) = \sqrt{\frac{a}{2g}} \int_{\theta_0}^{\pi} \sqrt{\frac{1 - \cos\theta}{\cos\theta_0 - \cos\theta}} d\theta$$
(12)

b. Use some trig identities and substitutions (i.e. try letting  $\phi = \theta/2$ ) to solve that integral and show that the answer is independent of position and only depends on a and g.

#### 2. Line on a Surface [A+C]

Consider a surface that is defined by the function:  $z = x^{3/2}$ .

a. Using the techniques of calculus of variations, find the curve that connects the two points: (0,0,0) and (1,1,1) with the shortest path.b. Make a 3d-plot showing this surface and the line from part (a). This <u>notebook</u> will be helpful.

#### 3. Simple projectile. [A]

Use the Lagrangian techniques to find the equations of motion for an object in 3 dimensions in a uniform gravitational field with no air resistance. Let z be the coordinate normally called 'up' and x and y can be situated in a plane parallel to the ground. (This is an easy one - just redoing basic kinematics using Lagrangians)

### 4. Spring/Pendulum [A]

Here is a pendulum + spring combination. There ball at the end of the spring has a mass m and the spring has a spring constant k. If nothing is attached to the spring, it will have a length l. The coordinate r is in the radial direction and has its origin at the top of the spring where it is attached to the support.

a. In the r and  $\theta$  coordinate system described above (please don't make up another one), write out the Kinetic and Potential energies for the pendulum. (Don't forget to include the potential energy of the spring, and make sure that it includes the length l so that when



Spring &

pendulum

r = l, the potential is zero. ) b. Apply the Euler-Lagrange equation to your Lagrangian L = T - U and obtain two equations of motion, one for r and another for  $\theta$ . (but don't solve for solutions.)

c. Setting all the acceleration and velocity variables equal to zero will yield the equilibrium conditions. You can see that  $\theta_0$  will be zero in equilibrium. What is the value of r in terms of m, g, l, and k?

d. Show how these equations are different than simple harmonic oscillators. What terms would have to be ignored (or considered small) to make them look more like SHO equations, i.e.  $\frac{d^2x}{dt^2} + a * x = 0$ ?

## 5. A dynamic ramp [A]



Lifting Ramp

Imagine a ramp that has the ability to increase its angle  $\theta$  w.r.t the ground. An object is at rest on the ramp, when the angle  $\theta$  is zero (that can be t = 0). The ramp is raised and the object begins to slide (no friction here). Using the Lagrangian techniques, find an equation for the position of the object along the surface of the ramp as a function of time. (You can call  $d\theta/dt = \omega$  and assume  $\omega$  to be constant.)

a. Find expressions for the kinetic and potential energies and build up a Lagrangian.

b. Use the Euler-Lagrange equation to find an differential equation for the acceleration of r.

c. Solve this non-homogenous equation to obtain a r(t) function.

\* This means that I'd like you to extend the techniques we use, and not use other software, libraries, etc. That means for plotting, use matplotlib, not plotly for example.