HW 1 PHYS 35100 solutions

Taylor

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

a) if $f(x) = \ln(1+x)$, evaluate at $a=0$

$$f(x) = \overset{x^0}{f(1+0)} + \dots \quad \overset{x^1}{f' = \frac{1}{1+0}} \; ; \; \overset{x^2}{f'' = -\frac{1}{(1+0)^2}} \; ; \; \overset{x^3}{f''' = \frac{1}{(1+0)^3}}$$

$$= \ln(1) + x - \frac{x^2}{2} + \frac{x^3}{3} + \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots$$

b) if $f(x) = \cos(x)$

$$\overset{x^0}{f(a) = \cos(0) = 1} \; ; \; \overset{x^1}{f'(a) = -\sin(0) = 0} \; ; \; \overset{x^2}{f''(a) = -\cos(0) = -1} \; ; \text{etc}$$

$$\therefore \; f(x) = 1 - \cancel{0x} - \frac{x^2}{2!} + \cancel{0\frac{x^3}{3!}}$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$$

c) if $f(x) = \sin(x)$

$$\overset{x^0}{f(a) = \sin(0) = 0} \; ; \; \overset{x^1}{f'(a) = \cos(0) = 1} \; ; \; \overset{x^2}{f''(a) = -\sin(0) = 0} \; ; \; \overset{x^3}{f'''(a) = -\cos(0)}$$

$$\therefore \; f(x) = \cancel{0} + 1x + \cancel{0\frac{x^2}{2!}} - 1\frac{x^3}{3!} + \dots$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$$

d) if $f(x) = e^x$ @ $a=0$

$$\overset{x^0}{f(0) = e^0} \; ; \; \overset{x^1}{f'(a) = e^0 = 1} \; ; \; \overset{x^2}{f''(a) = e^0 = 1} \; ; \; \overset{x^3}{f'''(a) = e^0 = 1} \quad \Big\} \text{all} = 1$$

$$\therefore \; f(x) = 1 + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + 1\frac{x^4}{4!}$$

$$\therefore \; e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$$