PHYS 351 - Fall 2021

Classical Mechanics

1. Classical Mechanics
2. What does "Classical" mean?
3. Time
4. The Outline
5. Tools

What does "Classical" mean?

We often use the adjectives *classical* and *quantum* to describe two different regimes of physics study. What's the difference?

How much do we know?

Where is this ball located right now? Be precise as possible.

What is its velocity? Be precise as possible.

"We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes."

Pierre Simon, Marquis de Laplace. (~1814)

The quote above is often refered to as "Laplaces Demon". It summarizes the idea of a **deterministic** universe, where, if we could only measure things precisely engough, and had unlimited computational power, we could predict the entirety of the universe's future. From his work: A Philosophical Essay on Probabilities

Example Problem #1:

A particle is moving in the x-direction and is confined between two perfectly bouncy (ahem, elastic) walls so that it bounces back and forth between $x=0$ and $x=l$, where l is the distance between the two walls. If there is an uncertainty in the initial velocity measurement, i.e. Δv_0, find the subsequent uncertainty in position at time t later.

The position of an object is given by:

$$x = v_0 t$$

If the velocity has an uncertainty in the initial measurement, then there is an uncertainty in position given by

$$\Delta x = (v_0 + \Delta v_0)\, t - v_0 t$$

which indicates that

$$\Delta x = \Delta v_0 \ t$$

This suggests that after a time $t_c = \frac{l}{\Delta v_0}$, whatever those numbers might be, the uncertainty of the particle will be equal to l, meaning its position is completely undetermined.

Example Problem #2:

Let a particle start from point A and move towards B. Assuming this is a perfect circle and that collisions with the walls are totally elastic, will it ever come back to point A? If so, how many bounces will it take?

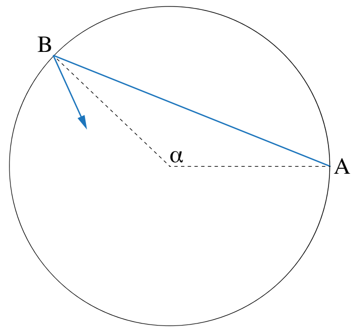

Fig. 1 A particle bounces off the inner walls of a circular constraint.

Consider the angle α as the some part of the whole circle, 2π. If it's one quarter of the circle (i.e. 90°), then it would be

$$\alpha = 2\pi\left(\frac{1}{4}\right)$$

thus:

$$4 \times \alpha = 2\pi \times 1$$

After 4 reflections, it will have gone around once. We can abstract this a bit and say that:

$$\alpha = 2\pi\frac{p}{q}$$

where $\frac{p}{q}$ is any rational number.

But what if $\alpha = 2\pi\times$ an irrational number?

Then, particle A will never return to the exact starting point. Interesting. One system is closed and periodic, the other is open and non-periodic. All just by changing the initial velocity.

Time

If we know how things move in the forward direction of time, can we also know how the process would happen in reverse?

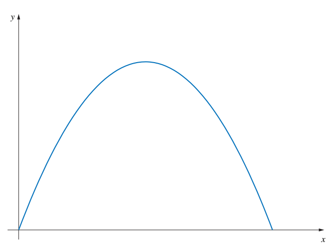

Fig. 2

The Outline

1. Newtonian Mechanics
2. Special Relativity
3. Variational Principle
4. Lagrangian & Hamiltonian Mechanics
5. Gravitation
6. Accelerating Reference Frames
7. Rigid Body Dynamics
8. Coupled Oscillators
9. ...and ?

Tools

- Everything from 20700 and 20800
- Advanced Calculus and other Math
- Computers

Intro Physics Review

Largely Mechanics (i.e. 20700) but with some references to electric fields and other 20800 material.

Example Problem #3:

> What are Newton's 3 Laws of motion?

Example Problem #4:

> What are the implications of the conservation of Energy?
>
> ... and Momentum?
>
> ... and Angular Momentum?

Example Problem #5:

> What goes on the R.H.S.?
>
> $$\frac{dU}{dx} = ?$$

Math you'll need

Differential Equations

$$\frac{dv}{dt} = \frac{F}{m}$$

But what if F is messy?

This is Newton's second law. It says the derivative of velocity w.r.t time is equal to the net force acting on the object divided by the mass. In intro physics, we mostly never had anything funny on the R.H.S. of the equation. It was generally a constant F. Now, we're going to be all sorts of things in that F. They might be time dependant, velocity dependant, etc. That makes solving the equation harder.

Taylor Series

The Taylor series is generally defined as: $f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$

For example, for the exponential function:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Vectors

To open the door, which direction should the torque vector point?

Linear Algebra

Computational Physics

Quick, you have 5 minutes to prepare a plot of:

$$U(x) = -\frac{1}{x}e^{-x} + \frac{1}{5x^2}$$

What do you do?