Homework 3

PHYS 351, Fall 2021

This set contains 4 problems. In general, show your work. Please do your best to make it readable and clear. If it's a huge mess, it will be harder to understand your efforts. Each problem is worth the same amount. Partial credit will be given, so please attempt them all.

DUE: Nov 10, 12:00 pm, on paper, in class.

1. Draw 3 Minkowski SpaceTime diagrams that include events and annotations/labels that clearly demonstrate the following implications from Special Relativity:

   a. That events that are simultaneous in one reference frame are not simultaneous in another inertial reference frame.

   b. The phenomenon known as *length contraction*

   c. The 'twin paradox' i.e. *time dialation*

   (Notes: you can either do this by hand with a ruler and make nice lines, or use a computer drawing application. You can also use this page to help guide you: https://sciencesims.com/sims/minkowski/)

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2. In the Brachistochrone Problem, we found the path of least time for an object sliding from a high point to a lower point to be a cycloid. Now, prove another interesting feature for object sliding down this curve: the time to reach the bottom is independent of the location on the cycloid curve.
   To do this, start with the following parametric equations for the cycloid:

$$x = \frac{a}{2}(\theta - \sin\theta) \text{ and } y = \frac{a}{2}(1 - \cos\theta) \tag{1.1}$$

   a. Show that the time to reach the bottom from an intial point B_0 is given by the integral:

$$t(B_0 \to B) = \sqrt{\frac{a}{2g}} \int_{\theta_0}^{\pi} \sqrt{\frac{1 - \cos\theta}{\cos\theta_0 - \cos\theta}} d\theta \tag{1.2}$$

   b. Use some trig identities and substitutions (i.e. try letting $\phi = \theta/2$) to solve that integral and show that the answer is independent of position and only depends on a and g.

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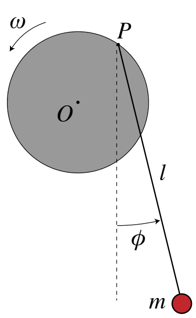
3. This figure shows a pendulum attached to a point on the rim of a rotating wheel. The string of length l is massless, and the bob has a mass m. The wheel is made to rotate at an angular speed ω.



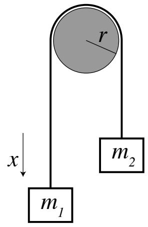
Pendulum Attached to a rotating wheel.

   a. Determine the Lagrangian for the pendulum.

   b. Use that to find and equation of motion for the angle ϕ. Your equation of motion should reduce to that of the simple pendulum:

$$\ddot{\phi} = -\frac{g}{l}\sin\phi$$

   in the case where $\omega = 0$

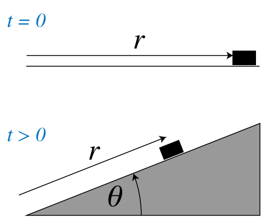


Atwood Machine

4. In the figure are two masses, m_1 and m_2, connected via a massless rope around a *massive* pully. The pulley has a mass of M and radius r.

a. Find the Lagrangian of this system

b. and then use it to find an equation for the linear acceleration of the two masses.



Lifting Ramp

5. Imagine a ramp that has the ability to increase its angle θ w.r.t the ground. An object is at rest on the ramp, when the angle θ is zero (that can be $t = 0$). The ramp is raised and the object begins to move. Using the Lagrangian techniques, find an equation for the position of the object along the surface of the ramp as a function of time. (You can call $\dot{\theta} = \omega$)
