Homework 2

PHYS 351, Fall 2021

This set contains ~~5~~ 4 problems. In general, show your work. Please do your best to make it readable and clear. If it's a huge mess, it will be harder to understand your efforts. Each problem is worth the same amount. Partial credit will be given, so please attempt them all.

DUE: September 29, 12:00 pm, on paper, in class.

1. **By Hand**: Using the Taylor Series expansions of $e^{i\theta}$ and $\sin(\theta)$ and $\cos(\theta)$, show how we can derive

$$e^{i\theta} = \cos\theta + i\sin\theta$$

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2. **By Hand**: For a simple harmonic oscillator, express A and ω in terms of two measurements you make of position and velocity, i.e. x_1 and v_1 and x_2 and v_2.

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3. **By Hand**: We solved the simple harmonic oscillator equation of motion in class by using

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt}$$

However, it can also be solved using the following approach: If the equation of motion is:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \tag{1.1}$$

then you can use a solution in the form of

$$x(t) = e^{qt} \tag{1.2}$$

and the task is to find q.

a. Put (1.2) in (1.1) and show how $q = \pm i\omega$ where ($i = \sqrt{-1}$)

b. This means the general solution will be a linear combination of two terms:

$$x(t) = a_1 e^{i\omega t} + a_2 e^{-i\omega t} \tag{1.3}$$

Find the two constant a_1 and a_2 in terms of the initial conditions: $x(0) = x_0$ and $v(0) = v_0$

c. Now, take (1.3) and show that this can be written as:

$$x(t) = a\cos(\omega t) + b\sin(\omega t) \tag{1.4}$$

where $a = a_1 + a_2$ and $b = i(a_1 - a_2)$

d. Lastly take (1.4) and show that this can be written as:

$$x(t) = A\cos(\omega t + \phi) \tag{1.5}$$

where $A = \sqrt{a^2 + b^2}$ and $\phi = -\tan^{-1}\frac{b}{a}$

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4. Hold off on this one for now. We'll come back to it later.
The equation of a pendulum of length L, without restricting its motion to small angles, is described by:

$$\ddot{\theta} + \omega^2 \sin\theta = 0$$

where $\omega = \sqrt{g/L}$. The pendulum is released from rest from an angle of

$$\theta_0 = \pi/2$$

. Pick a length L between 1 and 2 meters.

a. Numerically integrate this *true* equation of motion to obtain a table of data for $\theta(t)$. Show me how you did this.

b. Make a plot with two curves showing $\theta(t)$ for t from 0 to 10 s, one for the true eq. of motion listed above, and another that shows the equation of motion using the small angle approximation.

c. Compare the period of oscillations you discover to the approximation used for small angle oscillations.

(And we'll look at these numerical techniques again on either the 20th or 22nd so you don't need to freak out if you have no idea how to start this)

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5. Computational: In class, we'll look at the damped-driven pendulum demonstration. Using the parameters we figure out, prepare a mathematical model for the system. It should lead to a plot which roughly shows the observed motion of the system. (We'll talk more about this on Monday, 9/20)
