Homework 1

PHYS 351, Fall 2021

This set contains 6 problems. In general, show your work. Please do you best to make it readible and clear. If it's a huge mess, it will be harder to understand your efforts. Each problem is worth the same amount. Partial credit will be given, so please attempt them all.

DUE: September 13, 12:00 pm, on paper, in class.

1. Using the very basic algebra of the Galilean Transformations: at time t, find the length in the reference frame $\mathcal{O}$ of a meter stick that is at rest in the reference frame $\mathcal{O}'$. The meter stick has one end at $x' = 0$ and the other end at $x' = 1\ \mathrm{m}$. Do the same thing but for a meter stick oriented vertically, i.e. one end is at $y' = 0$ and the other end is at $y' = 1\ \mathrm{m}$. (And yes, this is the easy question of this homework set:) [MCM 1.1]

---

2. A force acts on a particle. It's a non-constant force.

   a. Here's the force function - it's *time* dependant:

$$F_x = F_0 + kt$$

   (F_0 and k are just positive constants, and t is time)
   Find expressions for the velocity and position as functions of time.

   b. Here's the force function - it's *position* dependant:

$$F_x = F_0 + kx$$

   (F_0 and k are just positive constants, and x is position)
   Find an expression for the velocity ~~and position~~ as a function of position.

---

3. Satellites in Low Earth Orbit (LEO) are traveling very quickly, at about 7500 m/s. But, there is still a little atmosphere present at their altitudes. The atmosphere gets thinner the higher up you go. We'll assume an orbital height of 550 km. Look up the atmospheric density at that height: http://www.braeunig.us/space/atmos.htm. Then use that in your calculations.

   a. Starting with $F = ma$, solve the differential equations of motion assuming a Netwonian Drag (i.e. quadratic dependence on v) force to figure out how long it would take a small satellite (i.e. a 1 m cube, with a mass of 200 kg), to decrease in speed by 1%, and 10%. (Keep this simple, assume the satellite is just traveling in a straight horizontal line though the upper atmosphere, don't worry about gravity, and orbits and all that. In reality, changing the orbital velocity will also change the orbital height which will change the air density and also the orbital speed, and so on... but let's hold off on those complications, for now.)

   b. Prepare a plot that shows this change in speed over time. Choose your units of time for the axis so that the graph communicates the interesting physics here.

---

4. A small spherical ball of mass m and radius R is dropped from rest into a vat of maple syrup, which of course has a large viscosity, η. The only appreciable forces on it are gravity mg and a linear drag force given by Stokes's law, $F_{\text{Stokes}} = -6\pi\eta R v$, where v is the ball's velocity, and the minus sign indicates that the drag force is opposite to the direction of v.

   a. Derive the velocity of the ball as a function of time (using analytical means).

   b. Then convince yourself (and me) that your answer makes sense for small times;

   c. ...and large times

   [MCM 1.10]

5. An object with mass m is attracted to the origin through an **inverse square law**, like this:

$$F(x) = -\frac{C}{x^2}$$

C is just a constant. It begins its motion from a position $x_0 = d$ with a velocity of zero.

a. Find an expression (using regular integration techniques, i.e. by hand, with just a pencil and paper) for the velocity as a function of position: $v(x)$
Hint: it should look something like :

$$v(x) = (\text{stuff})^{1/2}$$

b. What you've obtained in the first part is now a differential equation for $x(t)$. Use whatever method you want at this point to solve it to show that the time it takes to reach the origin is given by:

$$t = \pi\left(\frac{md^3}{8C}\right)^{1/2}$$

If you do use a symbolic computation method to solve it, indicate how you did it. Don't just copy the answer off the screen. Include some code or commands.

---

6. Using the results from the Example in class (quadratic Drag) problem in the class notes, compare the time it takes a 10 kg sphere vs a 1 kg to fall 50 meters in regular air. (You'll probably want to use a computer to help with this) Do you think this difference would be easy to see just by watching (i.e. not using any modern technology) ?
