#1 $bv + cv^2$

need to evaluate $\frac{cv^2}{bv}$ ... if large then ignore the linear term.

if small, then ignore the quadratic term.

The physical quantities we need to know are

η : viscosity of the fluid
R : radius of the object
v : speed of the object

for b

ρ : mass density of fluid
A : cross-sectional area
v : speed of object
C_D : Drag coefficient, i.e. shape

for c

#2 $x(t) = v_0 \tau \left(1 - e^{-t/\tau}\right)$

a) find $v(t)$ and $a(t)$

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}\left(v_0 \tau - v_0 \tau e^{-t/\tau}\right)$$

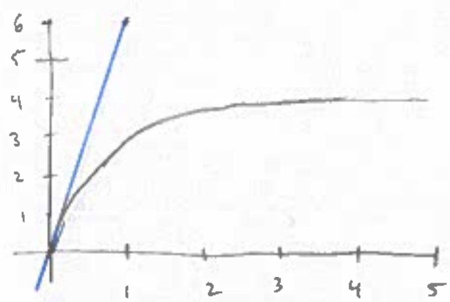
$$= -v_0 \tau \left(-\frac{1}{\tau}\right) e^{-t/\tau}$$

$$= v_0 e^{-t/\tau}$$

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(v_0 e^{-t/\tau}\right)$$

$$= -\frac{v_0}{\tau} e^{-t/\tau}$$

b)



if $b = 0 \Rightarrow \tau = \infty$

and the position vs. time would be a linear line : $x(t) = v_0 t$ from the slope we can say $v_0 = 6 \frac{mm}{s}$.

$\therefore v_0 \approx 6 \frac{mm}{s}$

c) as $t \to \infty$, $x(t) = v_0 \tau$

from the graph, the max of x is 4mm, so

$$4mm = 6 \, mm/s \cdot \tau$$

$$\therefore \tau = \frac{2}{3}$$

d) again, for $t \to \infty$,

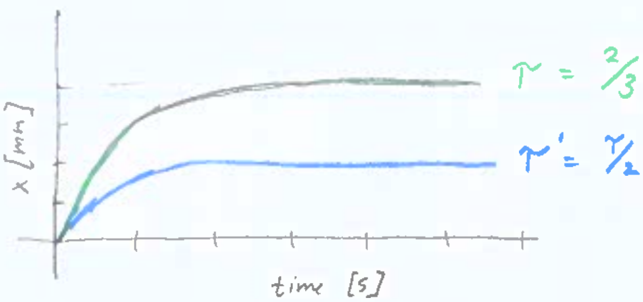
$$x = v_0 \tau$$

so if $b' = 2b$, then

$$\tau' = \frac{1}{2}\tau = \frac{1}{3}$$

and $x_{max}(\tau') = \frac{1}{3} \cdot v_0 = \frac{6\,mm}{3} = 2$

e)



#3 $F(v) = -\alpha v^{3/2}$

$F = ma$

a) $m\ddot{x} = -m\dot{x}^{3/2} = -\alpha v^{3/2}$

b) $m\dot{v} = m\frac{dv}{dt} = m\frac{dv}{dx}\frac{dx}{dt} = m\frac{dv}{dx}v = -\alpha v^{3/2}$

$$\frac{v}{v^{3/2}}dv = -\frac{a}{m}dx$$

$$-\frac{m}{a}\int \frac{1}{v^{1/2}}dv = \int dx$$ at $x = 0$, $v = v_0$; $x = x_{max}$, $v = 0$

$$-\frac{m}{a}\int_{v_0}^{0}\frac{dv}{v^{1/2}} = \int_0^{x_{max}} dx$$

$$-\frac{m}{a}\left[2v^{1/2}\right]_{v_0}^{0} = \left[x\right]_0^{x_{max}}$$

$$\frac{2m}{a}\sqrt{v} = x_{max}$$ ✓

#4

a) $\omega \to$ frequency of the driving force.

$\omega_0 \to$ fundamental, undamped frequency of the SHO $\to \sqrt{\frac{k}{m}}$

$\omega_1 \to$ damped frequency, i.e. the modified frequency due to the damping in the system

$$\omega_1 = \sqrt{\omega_0^2 - \beta^2}$$

b)
$$C = \frac{f_0}{\left((\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2\right)^{1/2}}$$

$$\begin{bmatrix} \text{find} \\ \omega \end{bmatrix} \frac{dC(\omega)}{d\omega} = \frac{d}{d\omega}\left(f_0\left((\omega_0^2-\omega^2)^2 + 4\beta^2\omega^2\right)^{-1/2}\right) = 0$$

all goes away

$$= \cancel{f_0}\left[\cancel{(\text{stuff})}^{-3/2}\left(2(\omega_0^2-\omega^2)(-2\cancel{\omega}) + 4\beta^2(2\cancel{\omega})\right)\right] = 0$$

$$-4(\omega_0^2 - \omega^2) + 8\beta^2 = 0$$

$$\therefore \quad \omega^2 = \omega_0^2 - 2\beta^2$$

$$\therefore \quad \omega_R = (\omega_0^2 - 2\beta^2)^{1/2}$$

c)
$$C(\omega_R) = \frac{f_0}{\left((\omega_0^2 - (\omega_0^2 - 2\beta^2))^2 + 4\beta^2(\omega_0^2 - 2\beta^2)\right)^{1/2}}$$

$$= \frac{f_0}{(4\beta^4 + 4\beta^2\omega_0^2 - 8\beta^4)^{1/2}}$$

$$= \frac{f_0}{(4\beta^2\omega_0^2 - 4\beta^4)^{1/2}} = \frac{f_0}{2\beta(\omega_0^2 - \beta^2)^{1/2}} = \frac{f_0}{2\beta\omega_1}$$ ✓

d) $\delta \to$ driving force can be out of phase of the system response.

(See fig 1.8 in MCM and p.20)