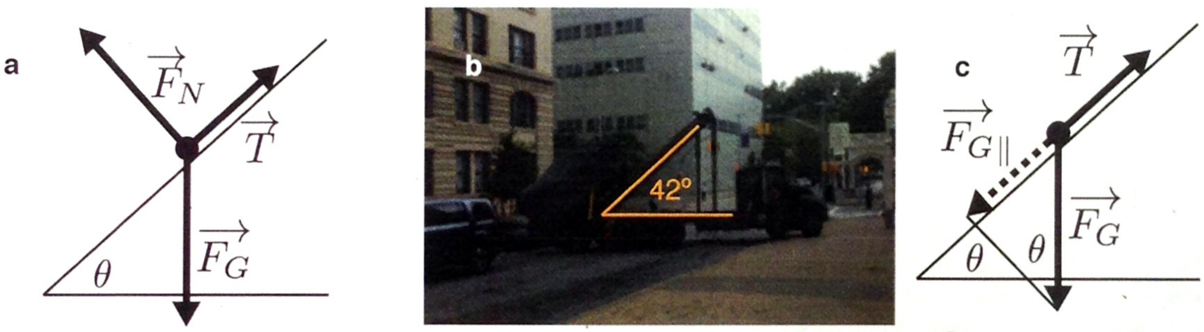
Find the Tension on the Cable.

By James A. Hedberg

Figure 1: The scene.

Here's a picture that shows a dumpster being lifted by a cable onto a flatbed truck. It was taken near Convent Ave and 140th street. Let's attempt to estimate the tension in the cable that is moving this dumpster up the truck's ramp. First, we'll need to make some assumptions about the situation.

We don't know the exact mass of the dumpster, but we can make some reasonable estimates. The type of dumpster appears to be a standard 10 yard roll-off container. These usually weigh about 2 tons, (1.5 tons empty, and let's assume 1000 lbs of garbage). (2000 lbs = 1 ton = 907 kg). [1]



Free Body Diagram

photograph analysis shows the angle of the ramp

Component of Gravity pointing parallel to the surface of the ramp.

Figure 2

To find the tension in the cable, let's first draw a free body diagram describing the situation. If we assume there is no friction between the dumpster and the truck, we will get a diagram like the one shown in figure 2a. The only forces acting on the dumpster will then be the force of tension from the cable, the force of gravity, and the normal force. We can use a commercial software application to measure the angle of the ramp, as shown in figure 2b. We measured it to be approximately 42° [2]. Using this number, we can then proceed to find the component of gravity parallel to the ramp, as shown in figure 2c. Trigonometric analysis of the vectors indicates the the component of gravity parallel to ramp will be given by

↙ should be the magnitude i.e. Absolute value.

$$\left|\vec{F}_{G\parallel}\right| = mg\sin\theta$$ [eq. 1]

Thus, with our assumed mass of 2 tons, which equals 1814 kg after converting to metric, we will obtain a value of 11,895 N. Thus, if the dumpster was just being held in place by the cable, the cable would have to have a tension in it equal to this, since the only two forces acting parallel to the ramp are the parallel component of gravity and tension. We can see this because of the sum of forces in the direction parallel to ramp should be zero, if there is no acceleration.

↙ vector.

$$\sum \vec{F}_{\parallel} = \vec{T} - \vec{F} = m\vec{a}_{\parallel} = 0$$ [eq. 2]

Another point of interest is exactly how the tension in the cable depends on the angle of the truck. In figure 3, we have plotted the tension in the cable, as predicted from the equation shown above, as a function of the truck bed angle. It makes sense that if the angle of the truck bed was zero, then there would be no tension in the cable. Likewise, if the angle of the truck were 90°, then the tension would be a maximum, given by the mass times 9.8 m/s^2.

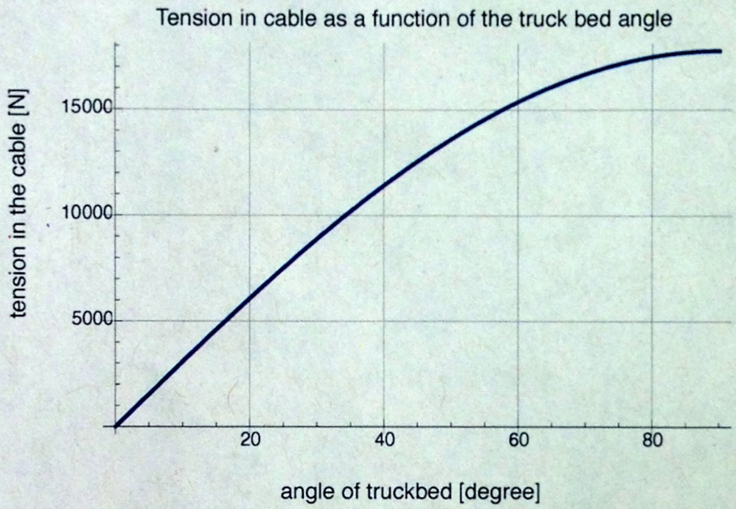


Figure 3

Further study of the system could include considering the friction between the dumpster and the truck, or how much work would be done by the motor in lifting the dumpster onto the bed. We could also figure out what types of cable would be able to support these tension. Surely, it would probably break if were made out of string. So, what should it be made out of? To answer this, we would need to consider the tensile strengths of the cable. These questions, while interesting and worthy of further study, are beyond the scope of this investigation.

References:

[1] Rudco Products. Roll-Off Containers. N.p.: Rudco Products, n.d. Web. 15 Oct. 2014. <http://www.rudco.com/pdf/rolloff_container8-14.pdf>.

[2] Adobe Illustrator was used to process the image and measure the angle.