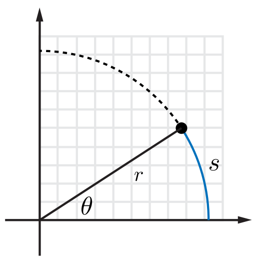
Rotational Kinematics

Introduction

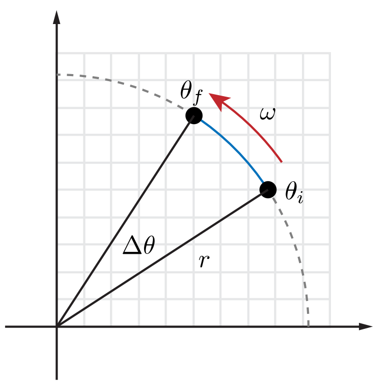


The goal: to re-express our kinematics, dynamics, and forces in terms of rotating objects.

We'll have to use angular displacements instead of linear ones.

This chapter is all about how to describe motion of objects undergoing rotation. The next chapter will deal the understanding the causes of rotation. This is distinction is similar to that between kinematics and Newton's laws. Kinematics told us how object moved, Newton's laws told us why they moved.

Rotational Motion Variables



Linear Velocity, v, referred to motion in$x$ or y directions (or a combination).

Angular velocity, ω, refers to the motion of an object around a center point.

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$

$$\bar{\omega} = \frac{\theta_f - \theta_i}{t_f - t_i}$$

$$\omega = \frac{d\theta}{dt}$$

This will be easier in the long run, although it takes some getting used to.

Example Problem #1:

> What is the angular velocity of the coin on the record player? (33.3 rotations per minute)

Angular Acceleration

Of course, ω can change.

If that happens then we'll have to use an angular acceleration to describe that motion.

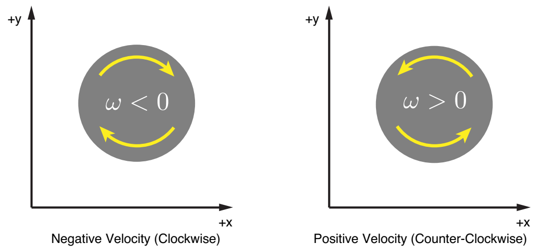
$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$$

$$\alpha = \frac{d\omega}{dt}$$

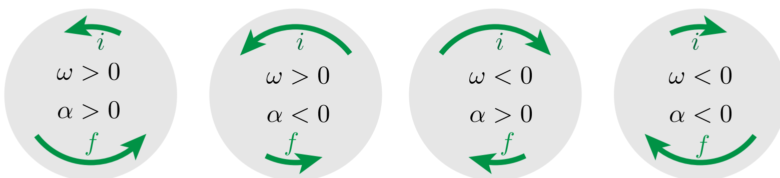
Sign Conventions

Choosing a positive and negative was easy in linear motion.

For angular velocities, we need a convention:

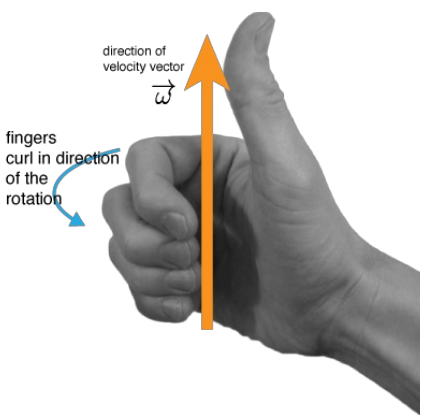


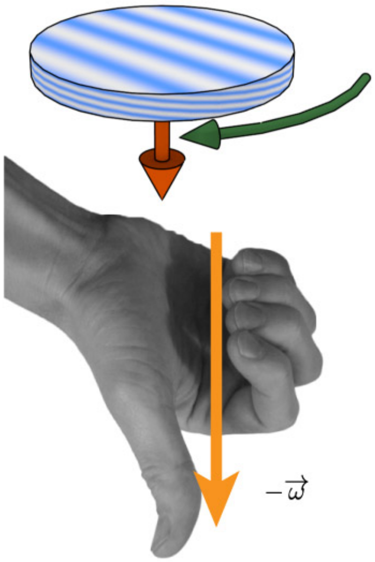
From this, we can also declare the sign of accleration:



Angular Velocity Vector

To describe the direction of an angular velocity, we'll need the**Right Hand Rule**.





In the case of a negative ω, the same right hand rule can help you out.

Rotational Kinematics

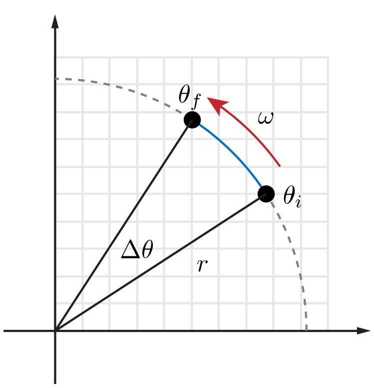
Fortunately, *they are all the same!*

Equations for linear motion	Equations for angular motion
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x = \frac{(v+v_0)t}{2}$	$\theta = \frac{(\omega+\omega_0)t}{2}$
$x = x_0 + v_0 t + \frac{at^2}{2}$	$\theta = \theta_0 + \omega_0 t + \frac{\alpha t^2}{2}$
$v^2 = v_0^2 + 2a_x x$	$\omega^2 = \omega_0^2 + 2\alpha\theta$

Must Use Radians!

Example Problem #2:

An old fashioned computer disk spins up to 5400 rpm in 2.0 s. Find the angular acceleration of the disc. After two seconds, how many revolutions has it made?



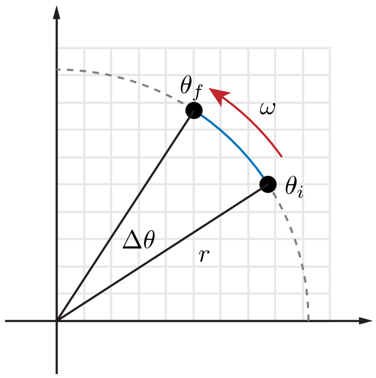
We can ask about the tangential velocity or speed of an object going around in circles.

Since $s = r\theta$ (in radians):

$$\frac{s}{t} = \frac{r\theta}{t} = r\omega$$

which will give us a tangential velocity: v_T.

$$v_T = r\omega$$

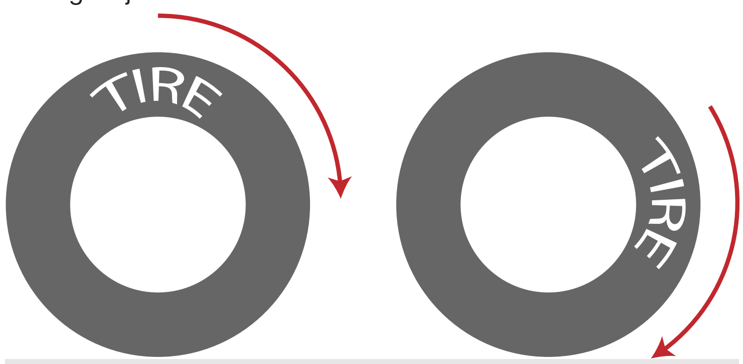


Similarly, we can ask about the tangential angular acceleration in the same way:

$$a_T = \frac{v_T - v_{T0}}{t} = \frac{r\omega - r\omega_0}{t} = r\left(\frac{\omega - \omega_0}{t}\right)$$

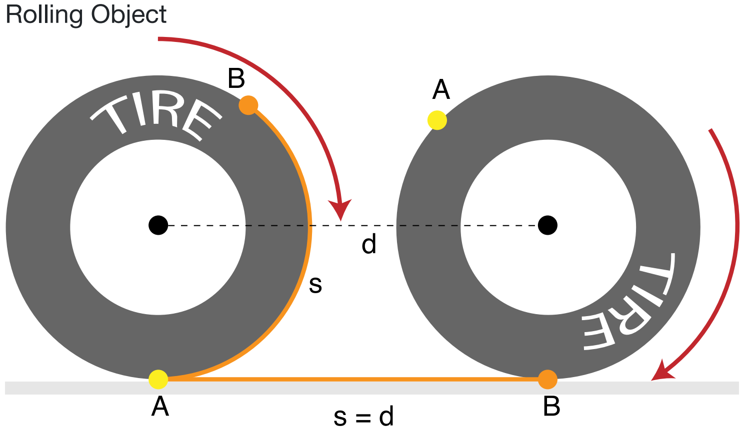
$$a_T = r\alpha$$

Rolling Objects



If an object is rolling, then it will undergo both rotational motion *and* translational motion.

Rolling Object



While the tire is rolling, the arc length s will be traced out along the ground.

If we consider the distance traveled by the tire, d, we can see that it must be the same as s. Thus:

$$v_{cm} = r\omega$$

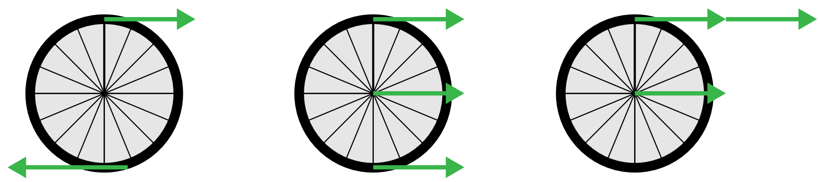
likewise for the acceleration:

$$a_{cm} = r\alpha$$

This condition is often called **rolling without slipping**.

Rolling without slipping

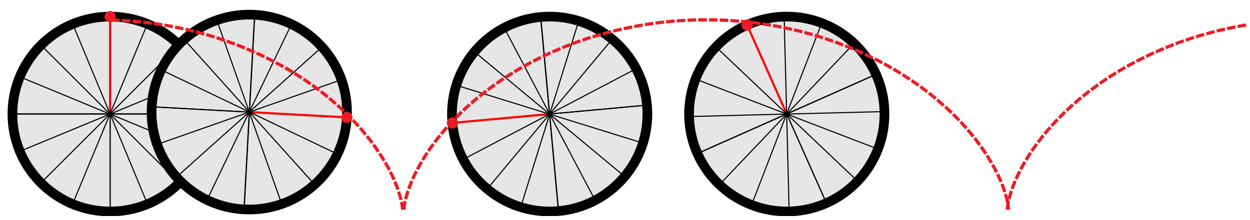
Vector addition still works



pure rotation pure translation rolling without slipping

Rolling

A point at the rim of a rolling wheel, when not slipping at all, will trace out the following curve:



It's known as a cycloid.

Angular momentum

Linear momentum was given by the mass times the velocity:

$$p = mv$$

Angular momentum will be given by the analogous terms:

$$L = I\omega$$

You'll note the letter I in the above equation for L, the angular momentum. This I is known as the moment of inertia and serves as the analogue for mass in rotational systems. We'll investigate it more in the next chapter.

The Cross Product

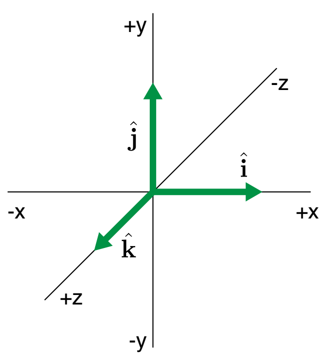
The cross product (or vector product) produces another vector:

2. The cross product (or vector product)

$|\mathrm{a} \times \mathrm{b}| = ab \sin\phi$

This produces a third vector that points perpendicular to both the original vectors.

$\mathrm{a} \times \mathrm{b} = (a_x \hat{\mathrm{i}} + a_y \hat{\mathrm{j}} + a_z \hat{\mathrm{k}}) \times (b_x \hat{\mathrm{i}} + b_y \hat{\mathrm{j}} + b_z \hat{\mathrm{k}})$



These are basic definitions of a right handed coordinate system:

$$\hat{\mathrm{i}} \times \hat{\mathrm{i}} = 0$$

$$\hat{\mathrm{j}} \times \hat{\mathrm{j}} = 0$$

$$\hat{\mathrm{k}} \times \hat{\mathrm{k}} = 0$$

$$\hat{\mathrm{i}} \times \hat{\mathrm{j}} = \hat{\mathrm{k}}$$
$$\hat{\mathrm{j}} \times \hat{\mathrm{i}} = -\hat{\mathrm{k}}$$
$$\hat{\mathrm{k}} \times \hat{\mathrm{i}} = \hat{\mathrm{j}}$$
$$\hat{\mathrm{i}} \times \hat{\mathrm{k}} = -\hat{\mathrm{j}}$$
$$\hat{\mathrm{j}} \times \hat{\mathrm{k}} = \hat{\mathrm{i}}$$
$$\hat{\mathrm{k}} \times \hat{\mathrm{j}} = -\hat{\mathrm{i}}$$

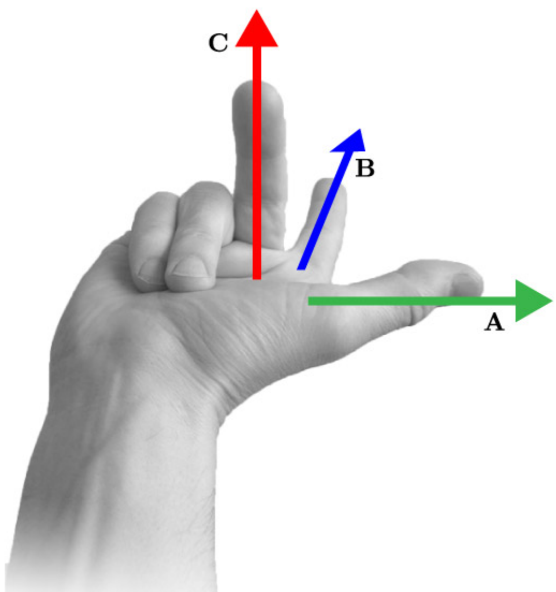
We can calculate the cross product by computing the determinant of this matrix:

$$\mathrm{A} \times \mathrm{B} = \begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

All said and done, it looks like this:

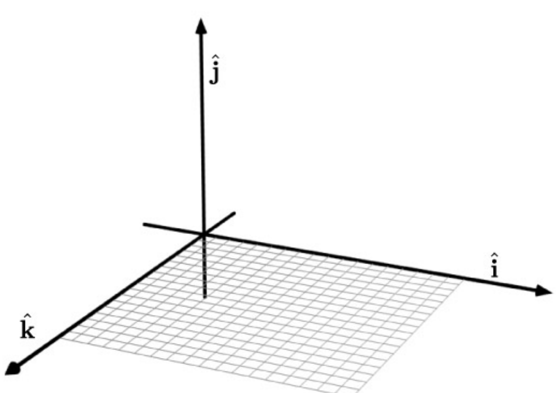
$$\mathrm{A} \times \mathrm{B} = (A_y B_z - A_z B_y)\hat{\mathrm{i}} + (A_z B_x - A_x B_z)\hat{\mathrm{j}} + (A_x B_y - A_y B_x)\hat{\mathrm{k}}$$

This is another vector!

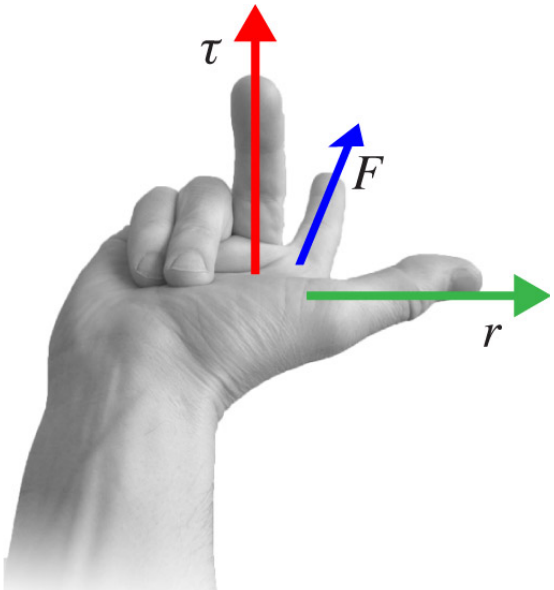


The **Right-Hand Rule** can be used to gain qualitative sense of direction when performing cross products.

$$\mathrm{A} \times \mathrm{B} = \mathrm{C}$$



$$\hat{\mathrm{i}} \times \hat{\mathrm{j}} = \hat{\mathrm{k}}$$



The **Right-Hand Rule, (version 2)** can be used to gain qualitative sense of direction when performing cross products.

Put your thumb in the direction of r, then your index finger in the direction of$F$. Torque will point in direction that your upright middle finger points.

The cross product: $\mathbf{A} \times \mathbf{B} = \mathbf{C}$

Example Problem #3:

If

$$\mathrm{A} = -2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$$

and

$$\mathrm{B} = -1\hat{\imath} + 1\hat{\jmath} + 2\hat{k}$$

find

$$\mathrm{A} \times \mathrm{B}$$

.

Example Problem #4:

Evaluate the following vector product:
If

$$\mathrm{A} = -1\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$$

and

$$\mathrm{B} = +3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$$

find

$$\mathrm{A} \times \mathrm{B}$$

.