Forces

Introduction

So far, we've just described how objects move, without much investigation into why.

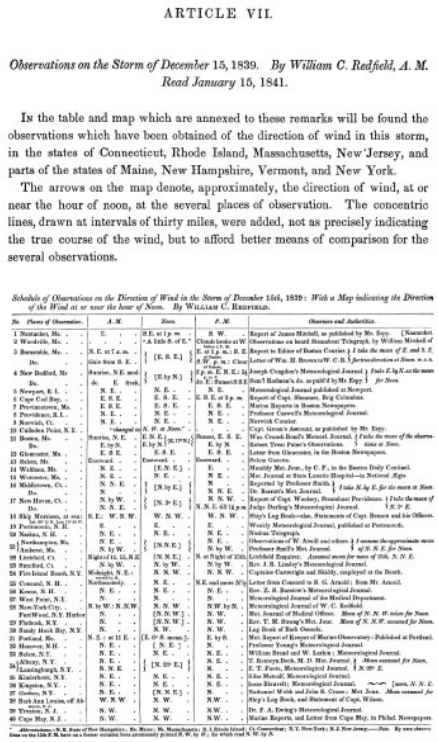
Science has to be more than just descriptive though.*

This really gets at the notion of science as a predictor. We need to be able to understand what will happen, not just what has happened.

*Unless you're in medicine, then you can just treat symptoms and ignore causes

Understanding causality is really one of the fundamental questions of science. At first, science consisted of merely describing the world around us. Naturalists could take note of what plants grew in certain areas at certain times and agricultural communities would use this information to help with crops. Astronomers would make very accurate celestial calendars that could be used for oceanic navigation. Doctors might notice that illness accompanied unclean conditions. But, there is a vast different between notices correlations and understanding causes.

Physics often strives to address the underlying causes for all phenomena. We are not happy knowing only when the sun will rise, but also why the sun rises, and why the time of the sunrise changes throughout the year.



ARTICLE VII.

Observations on the Storm of December 15, 1839. By William C. Redfield, A. M.
Read January 15, 1841.

In the table and map which are annexed to these remarks will be found the observations which have been obtained of the direction of wind in this storm, in the states of Connecticut, Rhode Island, Massachusetts, New Jersey, and parts of the states of Maine, New Hampshire, Vermont, and New York.

The arrows on the map denote, approximately, the direction of wind, at or near the hour of noon, at the several places of observation. The concentric lines, drawn at intervals of thirty miles, were added, not as precisely indicating the true course of the wind, but to afford better means of comparison for the several observations.

Schedule of Observations on the Direction of Wind in the Storm of December 15th, 1839: With a Map indicating the Direction of the Wind at or near the hour of Noon. By William C. Redfield.

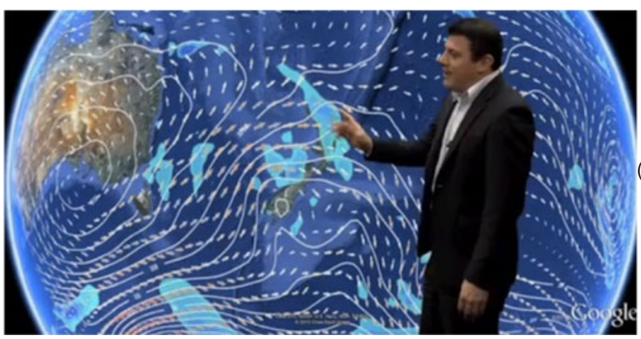
No. Places of Observation.	A. M.	Noon.	P. M.	Observers and Authorities.
1 Nantucket, Ms.	E.	S.E. at 1 p. m.	S. W.	Report of James Mitchell, as published by Mr. Espy. [Nantucket.
2 Woodville, Ms.	. . . .	"A little S. of E."	Clouds broke at W.	Observations on board Steamboat Telegraph, by William Mitchell of
3 Barnstable, Ms.	N. E. at 7 a. m.	[E. S. E.]	E. at 2 p. m.: S. E.	Report to Editor of Boston Courier. } I take the mean of E. and S. E.
Do.	Gale from S. E.		S.W. p. m.: Clear	Letter of Wm. H. Brown to W. C. R. } for true direction at Noon. w. c. r.
4 New Bedford, Ms	Sunrise, N.E. mod.	[E. by N.]	2 p. m. E. N. E.: 5½	Joseph Congdon's Meteorological Journal. } I take E. by N. as the mean
Do.	do. E. fresh.		do. E.: Sunset S.S.E.	Sam'l Rodman's do. as publ'd by Mr. Espy. } for Noon.
5 Newport, R. I.	N. E.	N. E.	N. E.	Meteorological Journal published at Newport.
6 Cape Cod Bay,	E. S. E.	E. S. E.	E. S. E. at 2 p. m.	Report of Capt. Slemmer, Brig Columbus.
7 Provincetown, Ms.	E. S. E.	E. S. E.	E. S. E.	Marine Reports in Boston Newspapers.
8 Providence, R.I.	N. E.	N. E.	N. E.	Professor Caswell's Meteorological Journal.
9 Norwich, Ct.	N. E.	N. E.	N. E.	Norwich Courier.
10 Culloden Point, N.Y.	"changed to	N. W. at Noon."	. . .	Capt. Green's Account, as published by Mr. Espy.
11 Boston, Ms.	Sunrise, N. E.	E. N. E. } [N. 17° E.]	Sunset, E. S. E.	Wm. Cranch Bond's Meteorol. Journal. } I take the mean of the observations at Noon.
Do.	E. by N.	E. by N.	E. by N.	Robert Treat Paine's Observations.
12 Gloucester, Ms.	E. S. E.	E. S. E.	E. S. E.	Letter from Gloucester, in the Boston Newspapers.
13 Salem, Ms	Eastward.	Eastward.	Eastward.	Salem Gazette.
14 Waltham, Ms.	N. E.	[E. N. E.]	E.	Monthly Met. Jour., by C. F., in the Boston Daily Centinel.
15 Worcester, Ms.	N. E.	N. E.	N. E.	Met. Journal at State Lunatic Hospital—in National Ægis.
16 Middletown, Ct.	N. N. E.	[N. by E.]	N.	Reported by Professor Smith. } I take N. by E. for the mean at Noon.
Do.	N.		N. N. E.	Dr. Barratt's Met. Journal.
17 New Haven, Ct.	N. by W.	[N. 3° E.]	N. N. W.	Report of Capt. Woolsey, Steamboat Providence. } I take the mean of N. 3° E.
Do.	N. N. E.		N. N. E. till 1½ p.m.	Judge Darling's Meteorological Journal.
18 Ship Morrison, at sea:	S. E.: W. N. W.	W. N. W.	W. N. W.	Ship's Log Book—also, Statements of Capt. Benson and his Officers.
19 Portsmouth, N. H.	E.	E.	E.	Weekly Meteorological Journal, published at Portsmouth.
20 Nashua, N. H.	N. E.	N. E.	N. E.	Nashua Telegraph.
21 Northampton, Ms.	N. E.	[N. N. E.]	N. E.	Observations of W. Atwill and others. } I assume the approximate mean of N. N. E. for Noon.
Amherst, Ms	N. by W.		N. by W.	Professor Snell's Met. Journal.
22 Litchfield, Ct.	Night of 14, 15, N.E.	[N. N. E.]	N. at Night of 15th.	Litchfield Enquirer. Assumed mean for noon of 15th, N. N. E.
23 Stratford, Ct.	N. by W.	N. by W.	N. by W.	Rev. J. R. Linsley's Meteorological Journal.
24 Fire Island Beach, N.Y.	Midnight, N. E.:	N. N. W.	N. N. W.	Captains Cartwright and Skiddy, employed at the Beach.
25 Concord, N. H.	Northeasterly.	N. E.	N.E. and more N'ly	Letter from Concord to S. G. Arnold; from Mr. Arnold.
26 Keene, N. H.	N. E.	N. E.	N. E.	Rev. Z. S. Barstow's Meteorological Journal.
27 West Point, N.Y.	N.	N.	N.	Meteorological Journal of the Medical Department.
28 New-York City,	N. by W.: N. N.W.	N. N. W.	N.W. by N.	Meteorological Journal of W. C. Redfield.
Fort Wood, N.Y. Harbor	N.	[N. N. W.]	N. W.	Met. Journal of Medical Officer. Mean of N. N. W. taken for Noon.
29 Flatbush, N.Y.	N.	[N. N. W.]	N. W.	Rev. T. M. Strong's Met. Jour. Mean of N. N. W. assumed for Noon.
30 Sandy Hook Bay, N.Y.	N.	N. W.	N. W.	Log Book of Bark Osceola.
31 Portland, Me.	N. E.: at 11 E.	[E. 6° S. mean.]	E. by S.	Met. Report of Keeper of Marine Observatory: Published at Portland.
32 Hanover, N.H.	N. E.	[N. E.]	N.	Professor Young's Meteorological Journal.
33 Salem, N.Y.	N. E.	N. E.	N. E.	William Bend and W. Larkin: Meteorological Journal.
34 Albany, N.Y.	N. E.	[N. 23° E.]	N. E.	T. Romeyn Beck, M. D. Met. Journal. } Mean assumed for Noon, N. 23° E.
Lansingburgh, N.Y.	N. N. E.		N.	E. T. Foote, Meteorological Journal.
35 Kinderhook, N.Y.	N. E.	N. E.	N. E.	Silas Metcalf, Meteorological Journal.
36 Kingston, N.Y.	N. E.	N. E.	N. E.	Isaac Blauvelt; Meteorological Journal. [noon, N. N. E.
37 Goshen, N.Y.	N. E.	[N. N. E.]	N.	Nathaniel Webb and John R. Crane; Met. Jour. Mean assumed for
38 Bark Ann Louisa, off Absecom, N. J.	W. N. W.	N. W.	N.W.	Ship's Log Book, and Statement of Capt. Wilson.
39 Trenton, N. J.	N. W.	N. W.	N.W.	Dr. F. A. Ewing's Meteorological Journal.
40 Cape May, N.J.	N. W.	N. W.	N.W.	Marine Reports, and Letter from Cape May, in Philad. Newspapers.

* Abbreviations.—N. H. State of New Hampshire; Me. Maine; Ms. Massachusetts; R. I. Rhode Island; Ct. Connecticut; N.Y. New-York; N.J. New Jersey.

Science for many centuries was just concerned with observing the natural world.

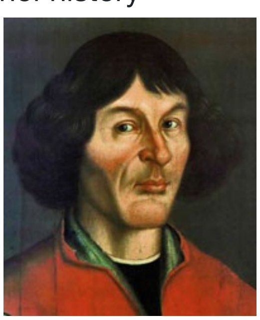
Observations on the Storm of December 15, 1839 Author(s): William C. Redfield Source: Transactions of the American Philosophical Society, New Series, Vol. 8 (1843), pp. 77- 82

We can do better.



(Vectors!)

A brief history

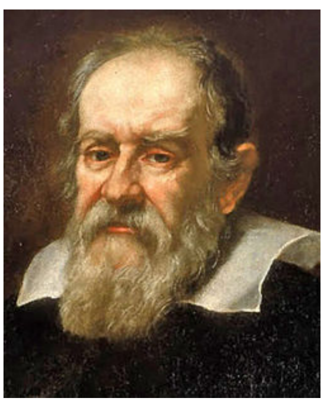


Nicolaus Copernicus got himself into trouble by saying the earth revolved around the sun. [1473 - 1543]

- 

Tycho Brahe took a lot of very nice data points about the positions of planets. [1546 - 1601]

Kepler, used Brahe's data to figure out some basic rules which described planetary motion quite well. [1571-1630]

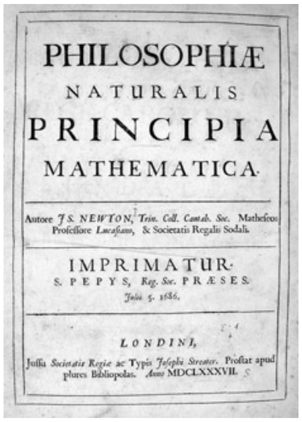
- 
Galileo had figured out $x \propto t^2$ for bodies under constant acceleration and that friction slows moving things down.[1564 – 1642]

Which brings us to 1680-1690

- The English had recently taken over a quant village called "New Amsterdam" from the Dutch.

- The last dodo bird was killed.
- Halley's comet was observed (and someone predicted its return. Guess who?)
- J.S. Bach was born
- Choclate Milk was invented
- and...

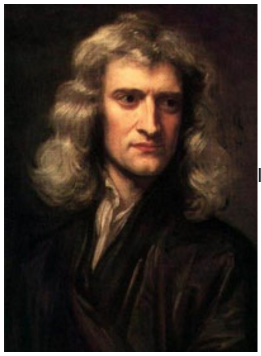
The Principia was published


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore J.S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.


Isaac Newton [1642-1727]

This work sought to unify the analysis of bodies in motion. That had not been done yet.

Should the planets and stars have a different set of laws which dictate their motion, compared to balls and stones flying around the earth's surface? No way!

The First Law

Every body continues in a state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.

If no net force acts on a body, its velocity won't change.

The first law describes what had been previously called *inertia*: the resistance of a physical body to a change in motion. Examples abound: rocks hurt when they hit you, it's hard to stop a car by standing in front of it etc.

Force

Force, at this point is not so easy to define in a nice, short, 'red box around it' kinda way. We could say some thing like: it's the cause of change in a bodies motion, or it's an influence on an object.

Instead, let's look at examples of forces.

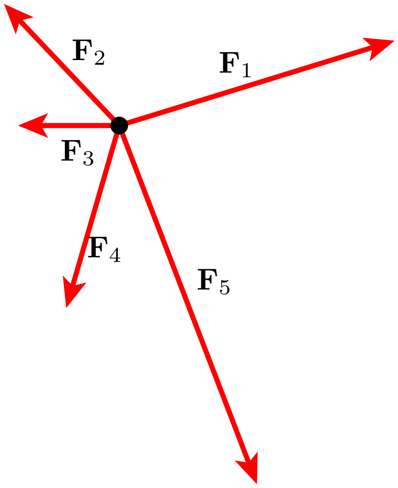
Classes of forces

1. **Contact Forces:** These forces involve physical contact between two bodies. Like pushing, pulling, rubbing, kicking, touching, etc.
2. **Fields:** A field can exert a force over a distance

How to describe a force

1. **A force is like a push or a pull:** The force is somehow responsible for the motion of another object
2. **A force acts on an object:** There was another object that moved. The force was applied to something.
3. **A force requires an agent:** There must be an 'agent' which acts on the object.
4. **A force is a vector:** A force has direction and magnitude, so it is certainly a vector quantity.

Multiple Forces



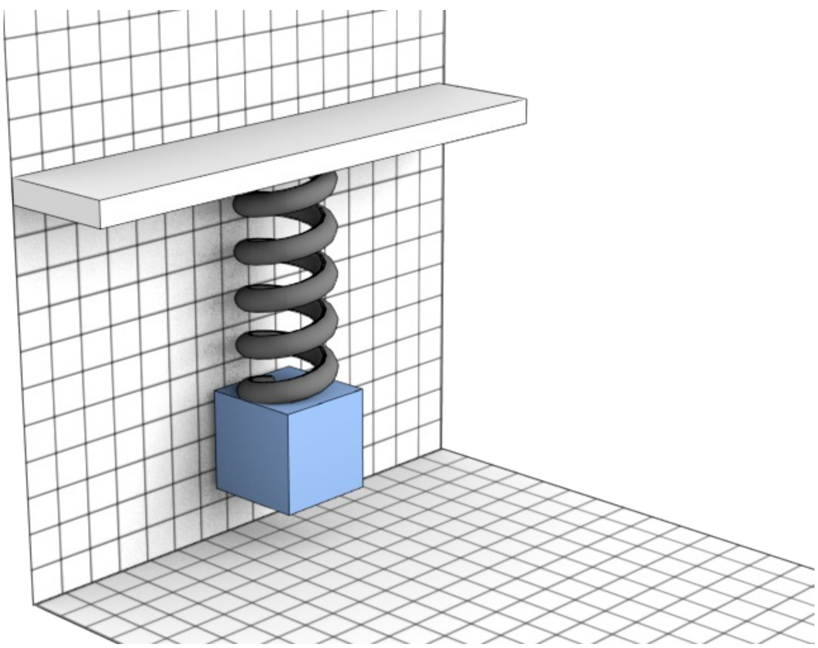
What about if we have multiple forces acting on an object.

Since we now know how to add vectors, we can handle this:

$$\sum \mathbf{F} = \mathbf{F}_{\text{net}} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 + \mathbf{F}_5 \ldots$$

It needs to be repeated, that all the same vector math we learned when dealing with velocities and accelerations, also applies to forces. Forces will have components, resultants, etc. They are treated in exactly the same way.

We can mix different forces:



Often, we'll have several different forces all acting on the same object.

For example, this box might have spring forces pulling it upwards, and weight pulling it downwards.

$$\sum \mathbf{F} = \mathbf{F}_{\text{spring}} + \mathbf{F}_{\text{weight}}$$

Newton's second law

The change in motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

The net force applied to a body produces a proportional acceleration in the same direction.

$$|\mathrm{F}| \propto |\mathrm{a}|$$

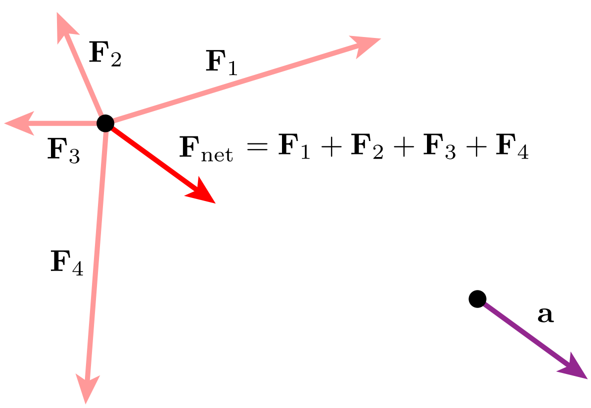
$$\mathrm{F} = m\frac{\Delta \mathrm{v}}{\Delta t} = m\mathrm{a}$$

The SI unit of force is called a Newton. We can see that is indeed made of a mass times and acceleration:

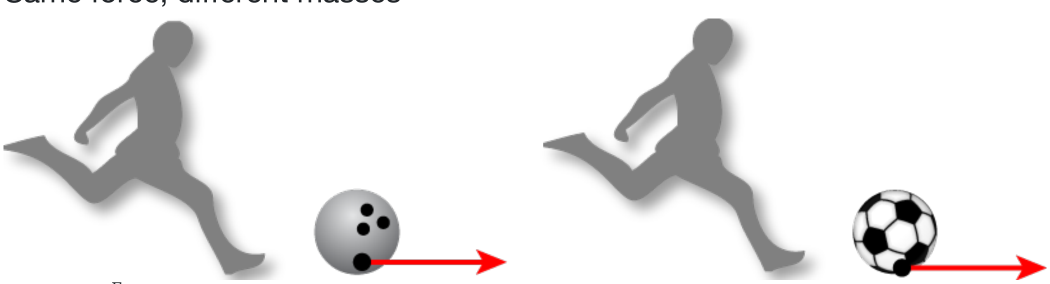
$$[kg] \times \frac{[m]}{[s]^2} = \frac{[kg][m]}{[s]^2} = 1 \text{ Newton}$$

Example Problem #1:

> The position of a particle along the x axis given by: $x(t) = 0.1t^3 - 2t^2 + 7t$ (distances are in meters). Its motion is caused by mysterious forces which are due to magical spells cast on the particle by several druids. At what time do the forces of each druid cancel each other to create no net force? Where along the x axis does this happen? Is the particle moving at this time? If so in which direction and how fast?



Same force, different masses



Since $a = \frac{F}{m}$, if we apply the same force to objects of different mass, we can see that the one with the larger mass will experience less acceleration.

For the bowling ball:

$$a_{\text{bowling ball}} = \frac{F_{\text{kick}}}{m_{\text{bowling ball}}}$$

For the soccer ball:

$$a_{\text{soccer ball}} = \frac{F_{\text{kick}}}{m_{\text{soccer ball}}}$$

Free body diagrams

One of our conceptual models we'll use frequently involves the creation of a *free body diagram*.

The free body diagram is a very powerful tool for reducing the complexity of physics problems. You really should practice drawing them. They will make this whole process easier.

The basic approach is to draw a dot that represents an object, then draw vectors on the dot that represent the magnitudes and directions of whatever forces **are acting on the object**.

Equilibrium

When, after adding together all the forces acting on an object, we find that the sum equals zero, then we'll say that the object, or system, is in *equilibrium*.

When the net force is equal to zero:

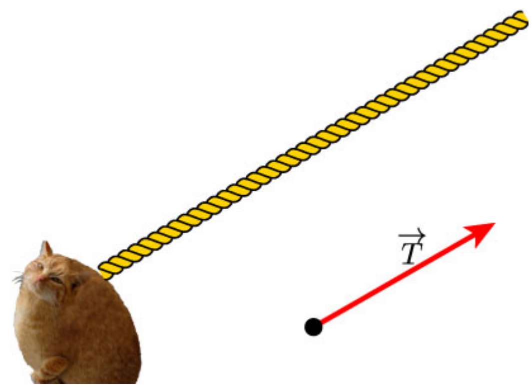
1. The acceleration is equal to zero
2. The velocity is constant

Equilibrium occurs when the net force is equal to zero.

1. The object, if at rest, will remain at rest
2. If the object is moving, it will continue to move at a constant velocity

A Catalog of Forces

Tensions

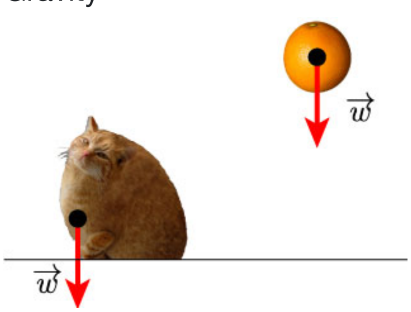


Tension force is a contact force exerted by a rope or string as it pulls on an object.

The direction is always in the direction of the pulling.

Here, we usually assume the rope is massless.

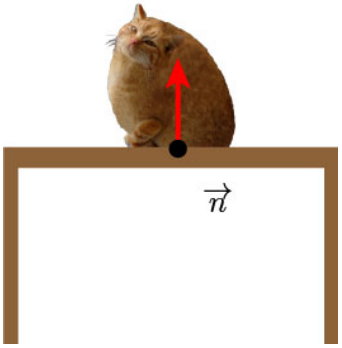
Gravity

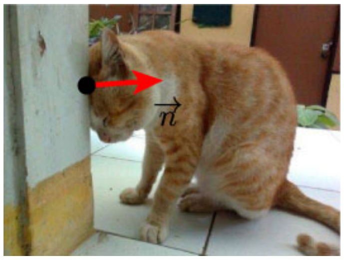


Gravity exerts a force on all massive objects. We call this force *weight.*

The Normal Force

The normal is a term from geometry meaning perpendicular. We call the force exerted by a surface on an object that is pressing against the surface.





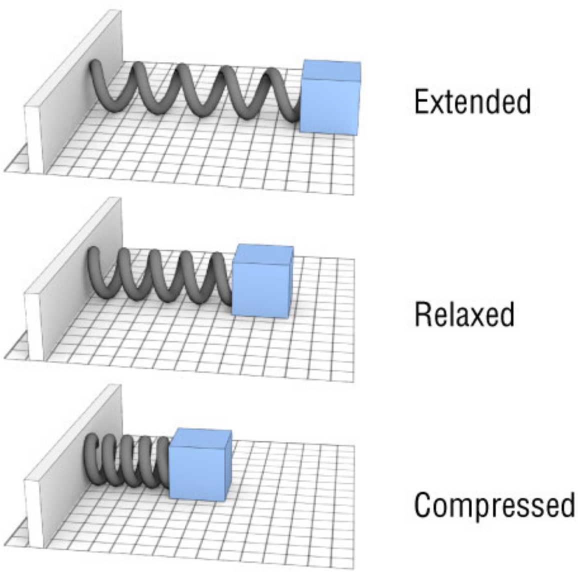
Friction

When an object slides, or attempts to slide over a surface, it experiences a resistance to movement, due to bonding forces between it and surface.

This resistance is frictional force f, directed opposite to the (attempted) motion.

Two kinds: kinetic (f_k) and static (f_s)

Spring or Restoring Forces



Spring math

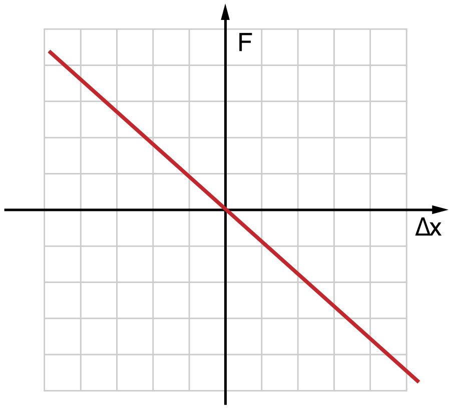
The force provided by a spring is proportional to its displacement.

$$F_{\text{spring}} = k\Delta x.$$

And, since force is a vector, we can write

$$\mathrm{F} = -k\Delta \mathrm{x}$$

Δx is our standard displacement and k is a constant called the *spring constant*. It tells us how stiff the spring is.



Drag

Another resistive force is drag. (Also called air resistance - what we've been ignoring)

Keep on ignoring it unless specifically told otherwise.

Small drag in streamlined position

Large drag in unstreamlined position

The Third Law

"To every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts."

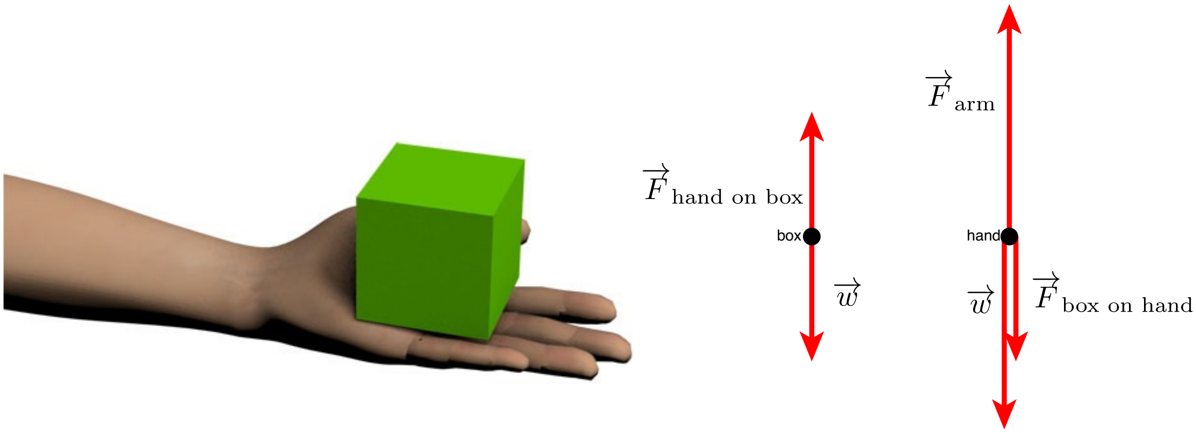
To every action there is always an equal and opposite reaction.

This is probably the most subtle of the laws. It's easy to think you understand it, but hard to master its implications.

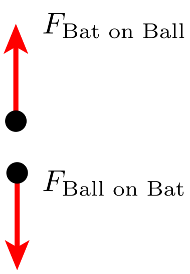
Forces, in the context of Newtonian mechanics, require an interaction between two objects. Thus, we have to use language like the following a lot: "The force of contact from object A on object B" or "The force of friction from the carpet acting on my knee". The third law says that the force from object A on object B will be the same magnitude as the force from object B on object A. (Say that out loud to yourself!) The only difference will be the direction of the vector.

Action/reaction pairs

$$\mathbf{F}_{\text{box on hand}} = -\mathbf{F}_{\text{hand on box}}$$



The bat on ball force gets all the credit. But, we mustn't neglect the force of the ball on the bat. That is really the essence of the third law.



Tactic: Identify the pairs:

The two forces in an action-reaction pair must act on different objects

The two forces in an action-reaction pair point in opposite directions and are equal in magnitude.

Newton's Laws

In words:

1. No change in velocity if there is no net force acting.
2. A force will cause a proportional acceleration inversely proportional to the mass.
3. Forces come in equal and opposite pairs

In symbols:

1. $\Delta v = 0$ if $F_{\text{net}} = 0$
2. F $= ma$
3. $F_{AB} = -F_{BA}$

A minor qualification

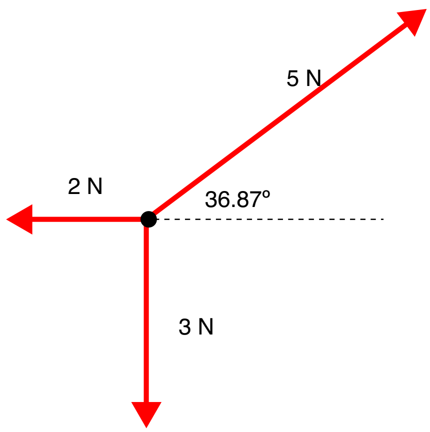
Newton's laws are only applicable in this form if the*system* we are observing is not accelerating.

Another way of saying this: We must be in an inertial reference frame.

Example Problem #2:

In a car crash, a car stops in 0.10 s from a speed of 14 m/s. The driver has a mass of 70kg. What is the force applied to the driver by the seat belt during the collision?

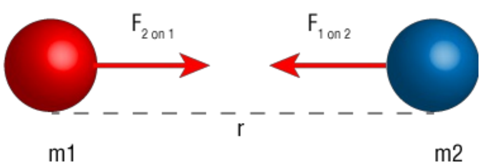
Example Problem #3:



Find the a_x and a_y for the object shown here. Its mass is 2.0 kg.

Gravity

Gravity is an attractive force between masses.



Its magnitude can be calculated by the following expression:

$$F_{\text{gravity}} = G\frac{m_1 m_2}{r^2}$$

G is a constant and equals $6.674 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$

Mass vs. Weight

Mass is an inherent property of an object:

a. Mass is a scalar quantity, with SI units of kg
b. Mass is independent of the object's surroundings
c. It is determined by the physical make up of the object.

Weight is equal to the magnitude of the gravitational force exerted on the object

a. An object's weight can be zero far from any gravitating body, but mass will not change
b. The same object can have different *weights* depending on where it is located.

(In English, we use these interchangeably -- I'll try to be careful)

Example Problem #4:

Find the gravitational force between two people sitting next to each other in this room.

Example Problem #5:

Find the gravitational force between the earth and a person.

Weight

The weight of an object on or above the earth is the gravitational force that the earth exerts on the object. The weight always acts downward, toward the center of the earth.

$$W = G\frac{M_E m}{r^2}$$

Here:
G, is the gravitational constant,
M_E is the mass of the earth, 5.98×10^{24} kg,
m is the mass of the object,
and r is the distance between the center of the earth and the object.

What are the units of weight?

Mass and Weight

We now have two equations for weight:

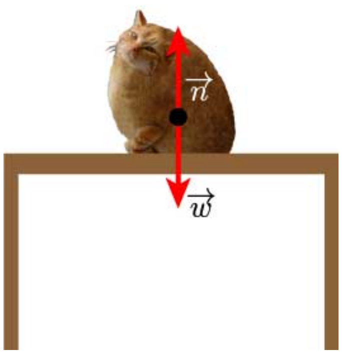
$$W = G\frac{M_E m}{r^2} = \left(G\frac{M_E}{r^2}\right) m$$

$$W = mg$$

Example Problem #6:

An astronaut's weight on earth is 800N. What is his weight on Mars, where the acceleration due to gravity there is 3.76 m/s^2?

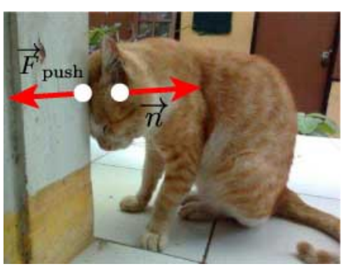
The Normal force



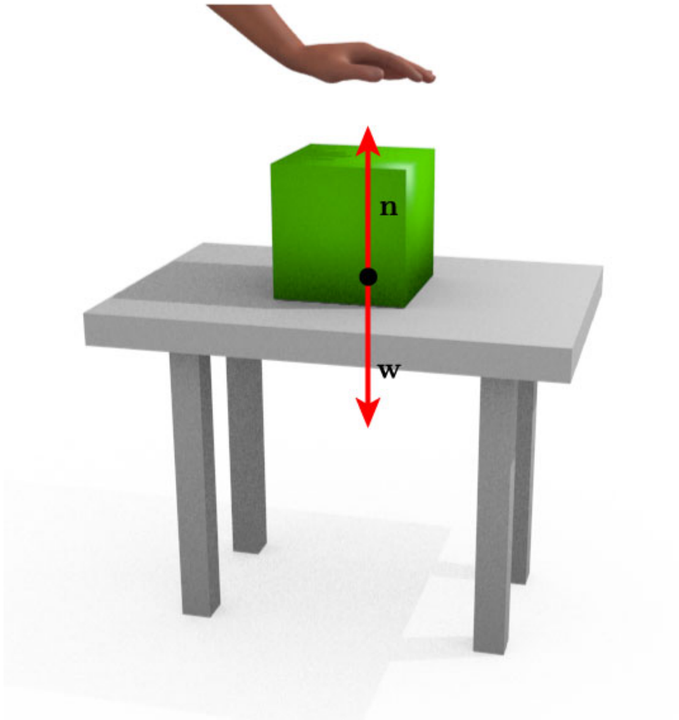
The magnitude of the normal force is equal to the weight force, and pointed in the opposite direction.

$$\vec{W} = -\vec{n}$$

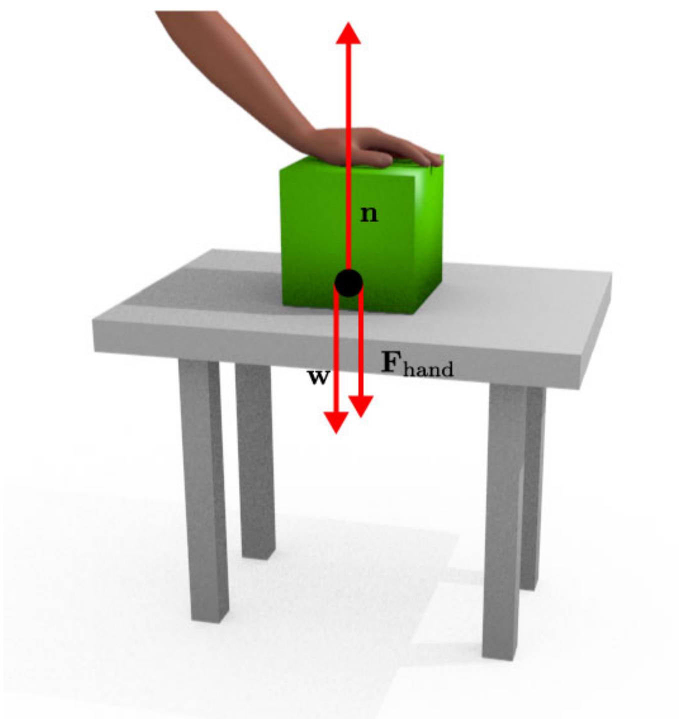
Here we have a cat on a table. There are two forces acting on the kitty: the weight (gravitational force) between the earth and the cat, and the normal force of the table on the cat.



This kitty is exerting a horizontal pushing force against the wall, and feels the normal force from the wall as a result.

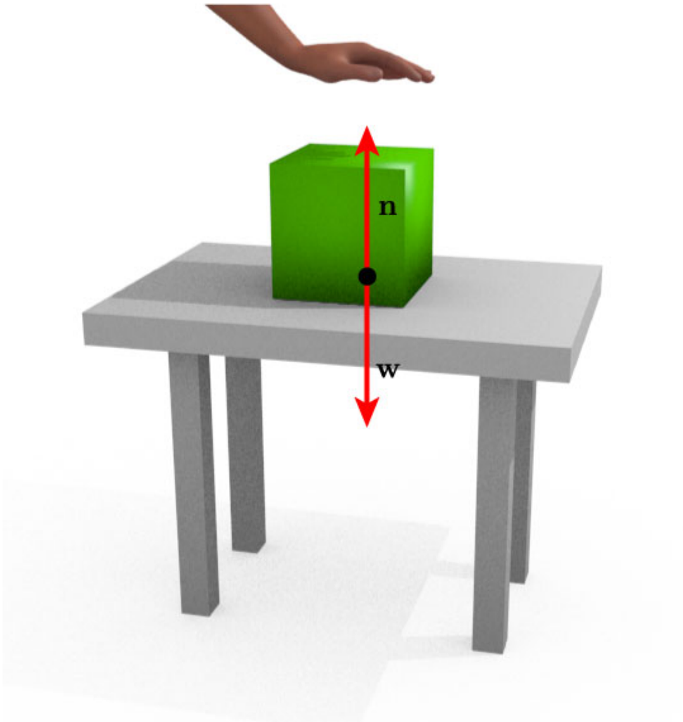


Here's a regular box on a regular table. The normal force is equal and opposite to the force due to gravity.

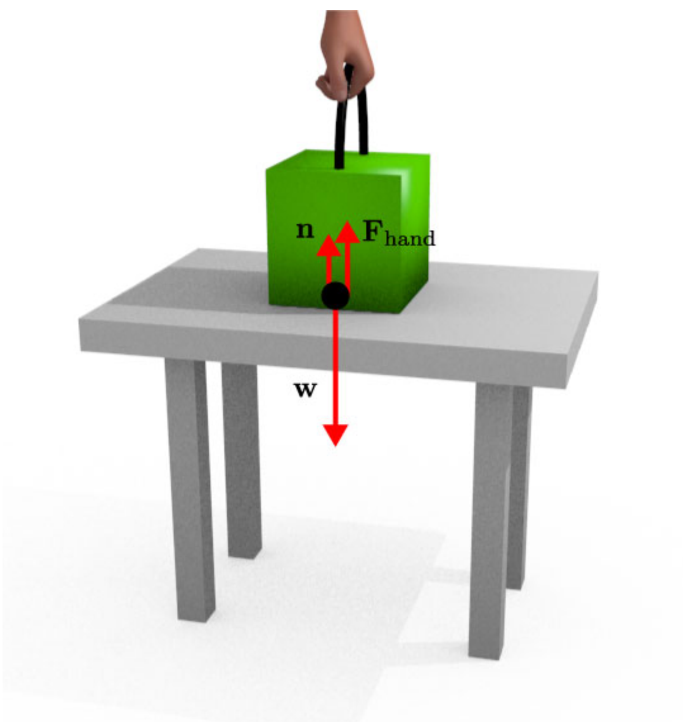


Now a hand applies a downward force to the box. The normal force increases to match the combined $\mathbf{F}_{\text{hand}}$ and $\mathbf{w}$.

$$\mathbf{n} = -(\mathbf{w} + \mathbf{F}_{\text{hand}})$$



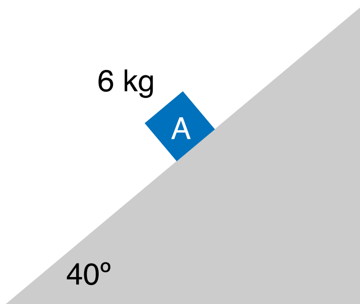
Here's a regular box on a regular table. The normal force is equal and opposite to the force due to gravity.



Likewise, if our hand force pulls the block up instead, this force will cause a decrease in the normal force

$$\mathrm{n} = -(\mathrm{w} - |\mathrm{F}_{\mathrm{hand}}|)$$

Example Problem #7:



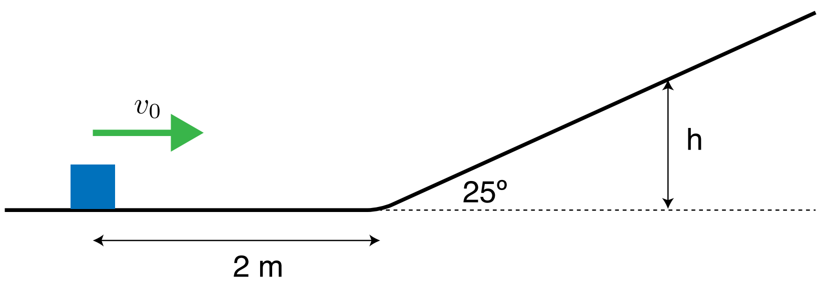
Draw a free body diagram for this box at it slides down this frictionless ramp. What is the magnitude of the normal force on the box? What is the acceleration of the box? Plot it as a function of time.

Example Problem #8:

If this 2kg box is moving towards the ramp on this frictionless surface, at 4 m/s:

(a) how long, starting from the position shown in the drawing, will it take to reach its maximum height?

(b) What is the height above the ground, h, where the block stops?



True and Apparent Weight

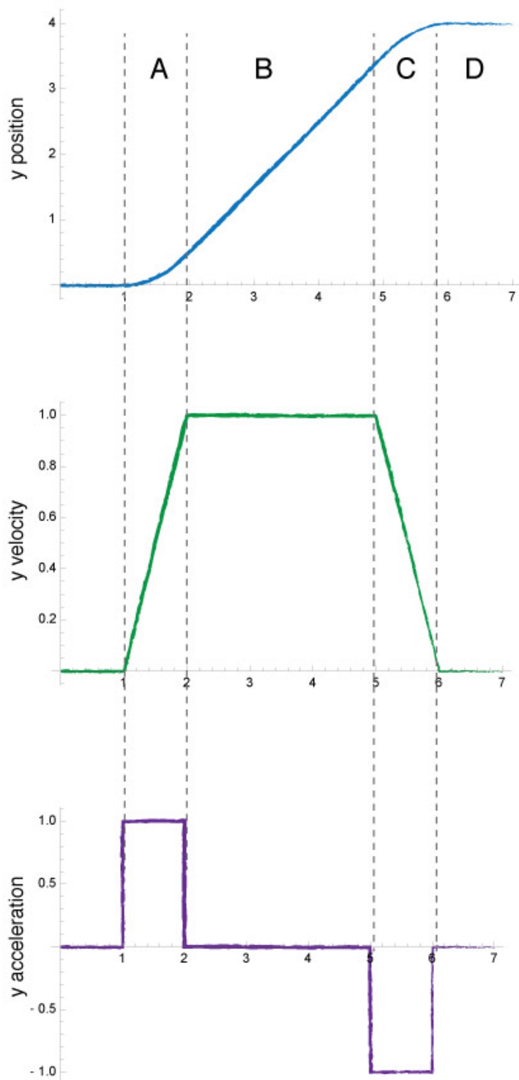
Our formula for calculating the weight of an object relies on the assumption that the object is not accelerating towards or away from the earth:

$$w = mg$$

However, if the object being weight is accelerating in the y axis, then we must account for that:

$$n_{\text{apparent}} = mg + ma$$

This is what happens if you try to use a scale while the elevator is accelerating up or down.

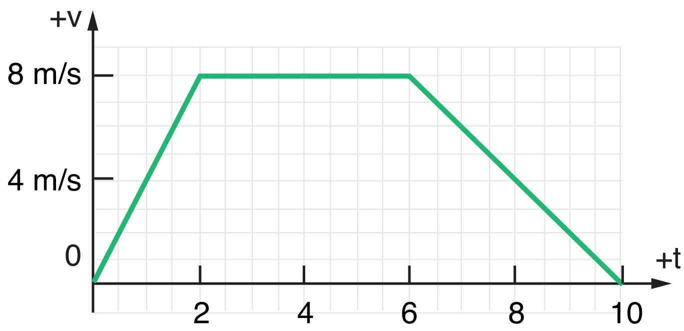


Here are plots of an object rising to a higher height on a elevator. There are 4 areas of interest in the plots.

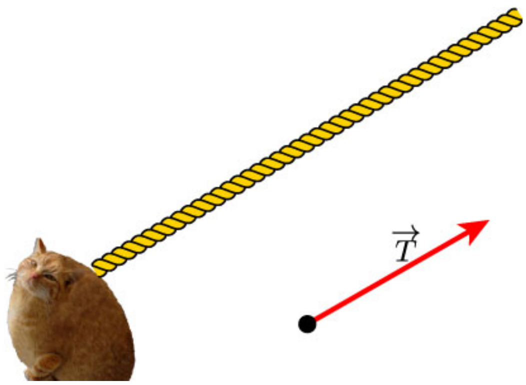
A. The object begins to accelerate in the positive direction. Its velocity starts increasing.
B. The object moves at a constant velocity in the positive direction. The acceleration is zero.
C. The object starts to slow down. Now we need an acceleration in the negative direction.
D. The object is at rest at the higher level.

Example Problem #9:

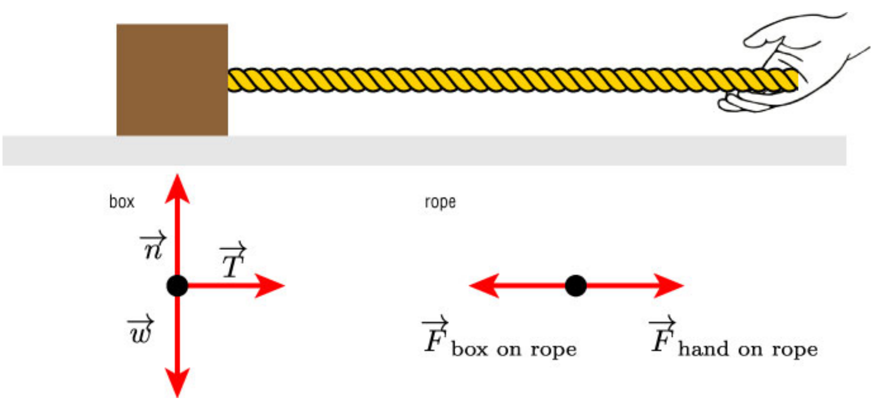
A 100 kg donkey rides in an elevator. Its velocity as a function of time is given by the following graph. What is the apparent weight of the donkey at $t = 1s, t = 5s$, and $t = 9s$?



Tension Force

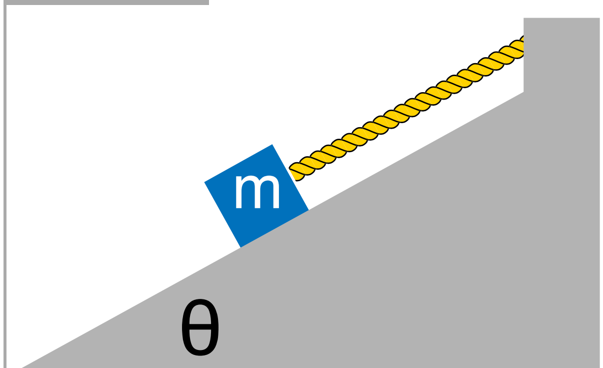


Ropes offer us a way to move objects. They do this by applying a tension force.



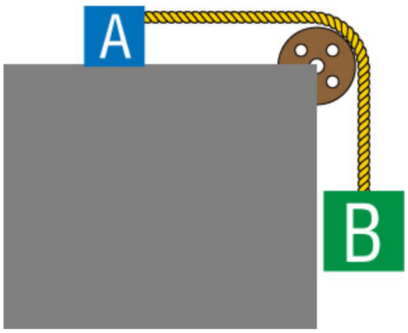
Here is a box that is pulled by a rope. The free body diagrams of the box and rope are shown.

Example Problem #10:



Find an expression for the tension in the rope, based on the relevant parameters.

Pulleys - what do they do?

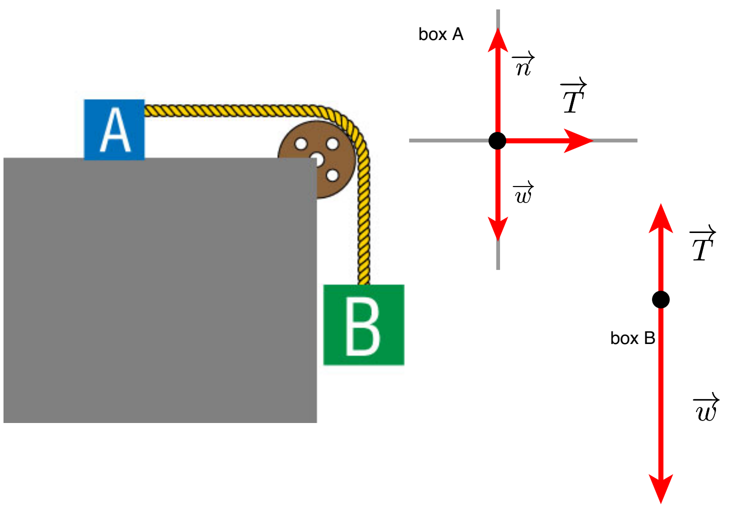


They redirect a force: The weight of Mass B is usually directed in the -y direction. However, by using a pulley, we can use the rope to accelerate Block A in the x direction.

Again: some assumptions:

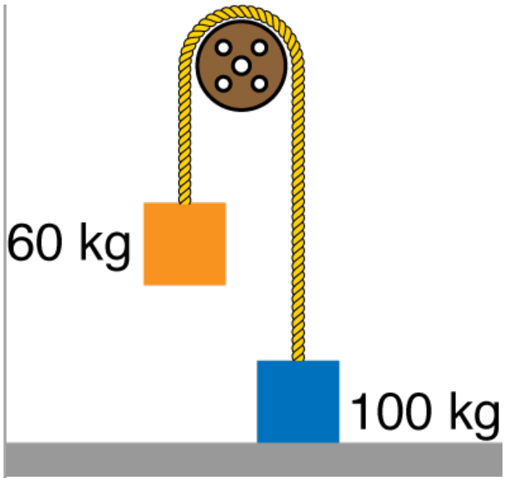
1. The pulleys we use are massless, just like the ropes.
2. A pulley is also frictionless on its axle.
3. Thus, none of the forces of a problem are ‘used’ to turn the pulley.

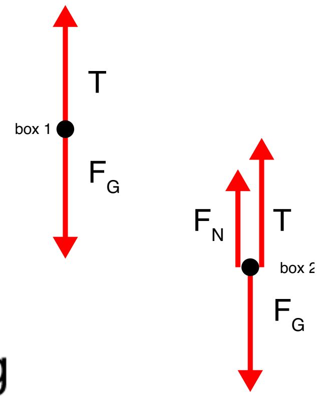
Free body diagram of the two objects.



Example Problem #11:

Find (a) the tension in the rope, and (b) the normal force acting on the blue box.





We can start by considering the *sums of forces* on each box (in the y direction only, since there is nothing interesting happening in the x). The first box has a Tension force pointing up (+) and a gravitational force pointing down (-). Together these equal its mass times its acceleration:

$$\sum F_y^1 = +T - F_G = m_1 a_1$$

The second force will have Tension, gravity, and also a normal force. This normal force is pointing up (+).

$$\sum F_y^2 = +T + F_N - F_G = m_2 a_2$$

We also know that since the objects are not accelerating, that both of these force summations will equal 0.

$$T - m_1 g = 0 \quad (1)$$
$$T + F_N - m_2 g = 0 \quad (2)$$

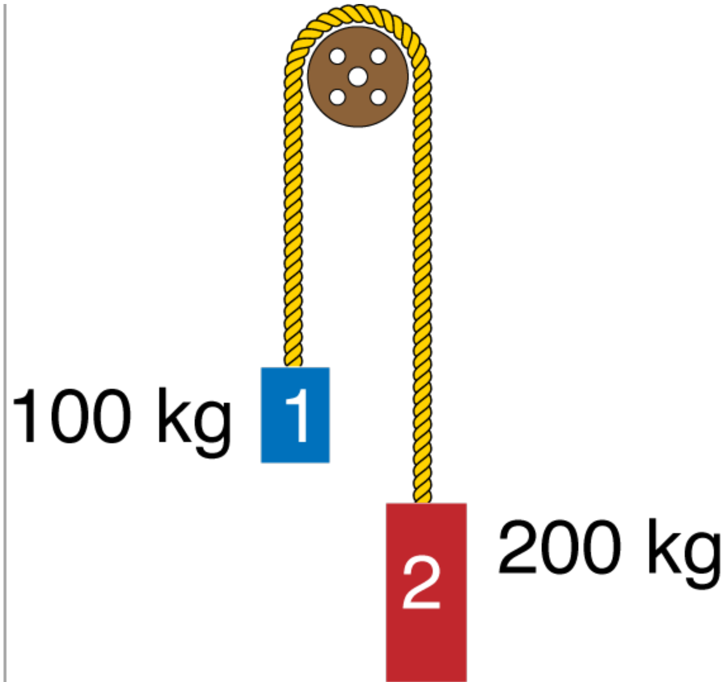
Part a) wants us to get the Tension. We can solve for that directly:

$$T = m_1 g = 60 \text{ kg} \times 9.8 \text{ m/s}^2 = 588 \text{ N}$$

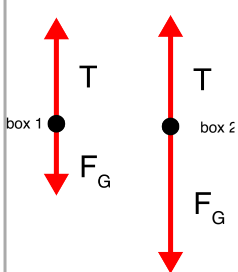
Part b) needs the normal force. We can combine the two equations above to get an expression for the normal force:

$$F_N = m_2 g - T = m_2 g - m_1 g = (m_2 - m_1) g = (100 \text{ kg} - 60 \text{ kg}) \times 9.8 \text{ m/s}^2 = 392 \text{ N}$$

Example Problem #12:



Mass 2 is allowed to fall. It is connected via the rope and pulley to mass 1. What is the acceleration of mass 1?



We start this problem by writing the *sums of forces* for both blocks, after consulting our free body diagrams.

$$\sum F_y^1 = +T - F_G = m_1 a_1$$

$$\sum F_y^1 = +T - F_G = m_2 a_2$$

Since each box has a different mass, the force of gravity will be different on each:

$$F_G^1 = m_1 g$$

$$F_G^2 = m_2 g$$

We also know that since they are connected by the rope, that their accelerations will be the same magnitude, but pointed in opposite directions. (Box 2 will accelerate down, box 1 will accelerate up.) This allows us to write the constraint equation:

$$a_1 = -a_2$$

Now, we can use this to reduce the number of variables.

$$T - m_1 g = m_1 a_1 \tag{1}$$
$$T - m_2 g = m_2 a_2 = m_2(-a_1) \tag{2}$$

We need to solve these simultaneously for a_1. One method for that is by eliminating T, the tension force. Rearrange the first eq (1).

$$T = m_1 a_1 + m_1 g$$

Putting this into equation (2) yields:

$$\overbrace{m_1 a_1 + m_1 g}^{T} - m_2 g = m_2(-a_1)$$

Let's bring the a_1 terms to the L.H.S and the g terms to the R.H.S.

$$m_1 a_1 + m_2 a_1 = m_2 g - m_1 g$$

Pull out the a_1 and g

$$a_1(m_1 + m_2) = g(m_2 + m_1)$$

Solve for a_1

$$a_1 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

If $m_1 = 100$ kg and $m_2 = 200$ kg, this reduces to:

$$a_1 = \frac{1}{3} g = 3.27 \text{ m/s}^2$$

Things to note: if $m_2 = 0$, then the acceleration of m_1 will just be $-g$, which is what we'd expect. Yay.

Equilibrium Applications

Equilibrium for an object or system of interacting objects occurs when there is no acceleration of any of the objects.

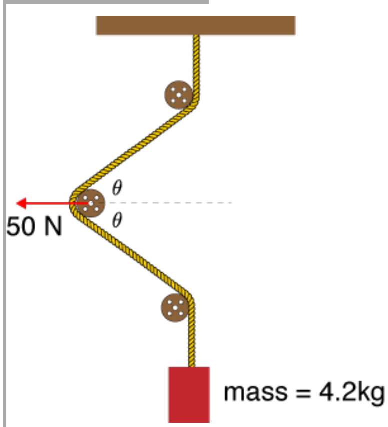
$a_y = 0$, and $a_x = 0$

This implies the net forces in both directions are all zero as well:

$\sum F_x = 0$ and $\sum F_y = 0$

This *does not imply* that the velocity of the object(s) has to be zero.

Example Problem #13:



What is the angle θ in this diagram? The mass and the pulleys are all stationary.

Non-equilibrium Applications

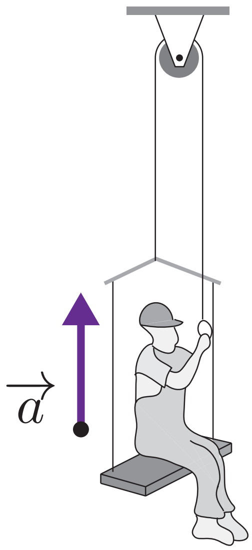
In the case that there *is* an accelerating object, then we know the net force in that direction is *not* zero Newtons.

Instead of setting the net force equal to zero, it will equal the acceleration in that direction:

$$\sum F_x = F_{\text{net}_x} = m a_x$$

$$\sum F_y = F_{\text{net}_y} = m a_y$$

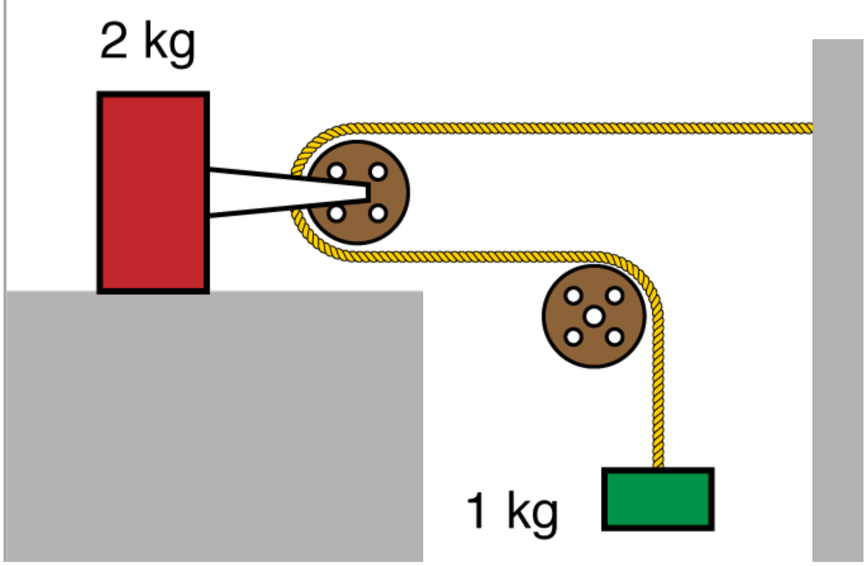
Example Problem #14:



A painter uses the chair and pulley arrangement as shown to lift himself up the side of the house. The painter's mass is 70 kg and the chair's mass is 10 kg. a) With what force must he pull down on the rope in order to accelerate upward at 0.2 m/s^2? (The pulley is frictionless and massless

Example Problem #15:

Find the acceleration of the 2 kg block.



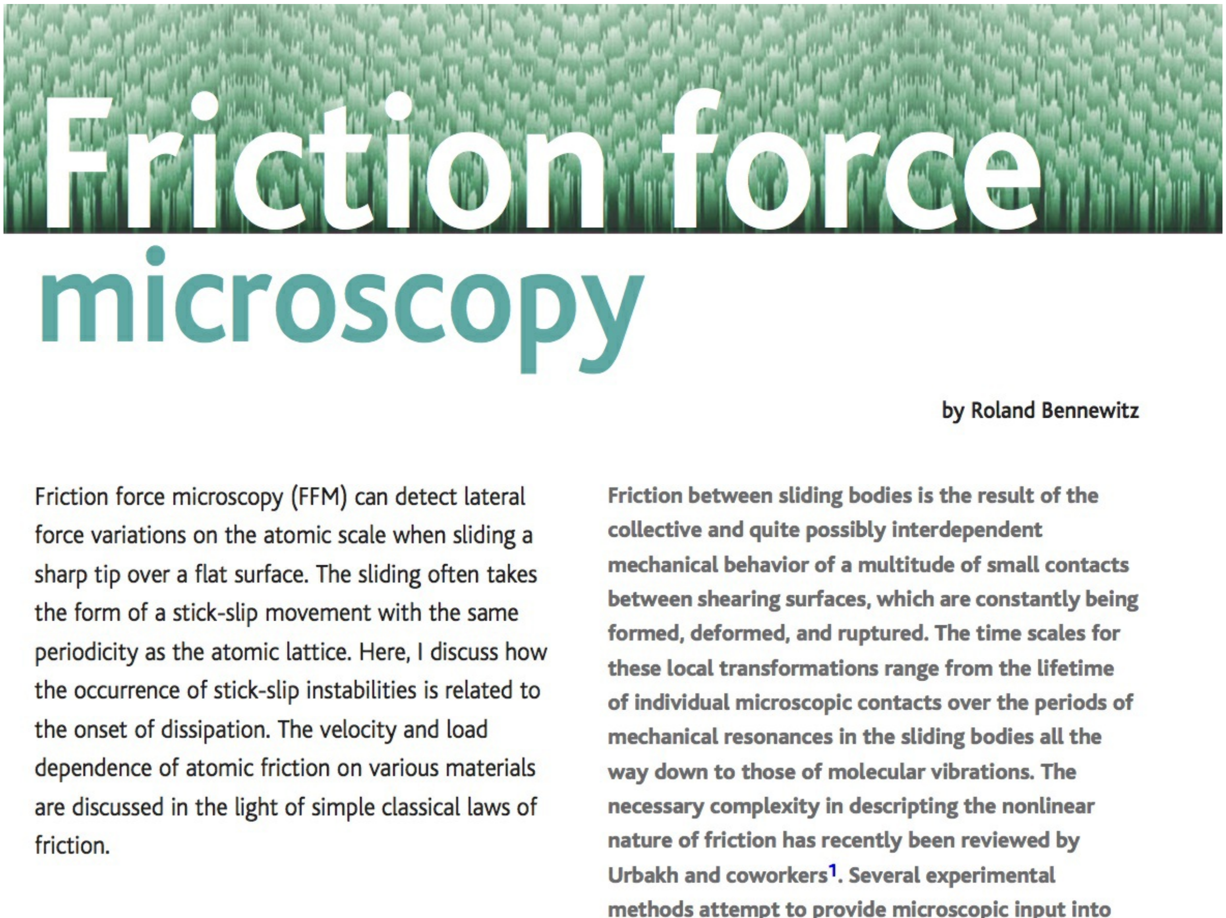
Example Problem #16:

A skier slides down a slope of 10 degrees. A wind blows along the direction of the slope. Let's say the skier has a mass of 40 kg. Find the acceleration of the skier as a function of the wind force. Sketch a plot of this function.

friction, we would have to account for atomic scale forces and interactions. That's hard!
a model of the way friction forces work.

Example Problem #17:

An elevator cab and its load have a combined mass of 1600 kg. Find the tension in the supporting cable when the cab, originally moving downward at 12 m/s, is brought to rest with constant acceleration in a distance of 42 m.

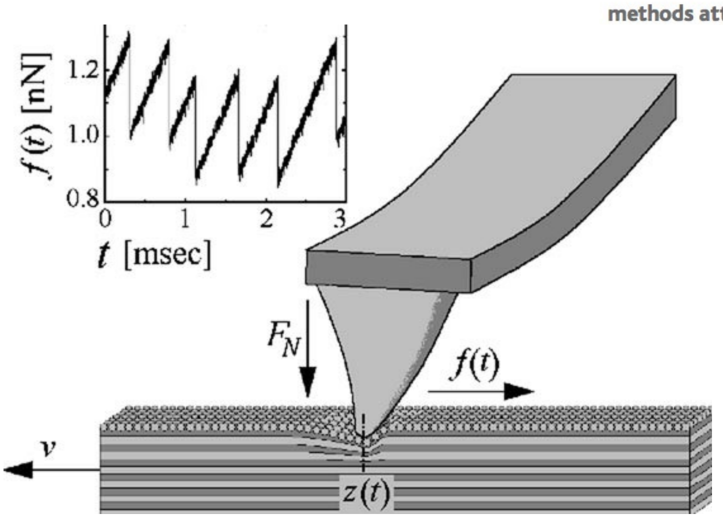


Friction force microscopy

by Roland Bennewitz

Friction force microscopy (FFM) can detect lateral force variations on the atomic scale when sliding a sharp tip over a flat surface. The sliding often takes the form of a stick-slip movement with the same periodicity as the atomic lattice. Here, I discuss how the occurrence of stick-slip instabilities is related to the onset of dissipation. The velocity and load dependence of atomic friction on various materials are discussed in the light of simple classical laws of friction.

Friction between sliding bodies is the result of the collective and quite possibly interdependent mechanical behavior of a multitude of small contacts between shearing surfaces, which are constantly being formed, deformed, and ruptured. The time scales for these local transformations range from the lifetime of individual microscopic contacts over the periods of mechanical resonances in the sliding bodies all the way down to those of molecular vibrations. The necessary complexity in descripting the nonlinear nature of friction has recently been reviewed by Urbakh and coworkers[1]. Several experimental methods attempt to provide microscopic input into



Europhys. Lett., 67 (6), p. 907 (2004)

Static Friction

Some rules:

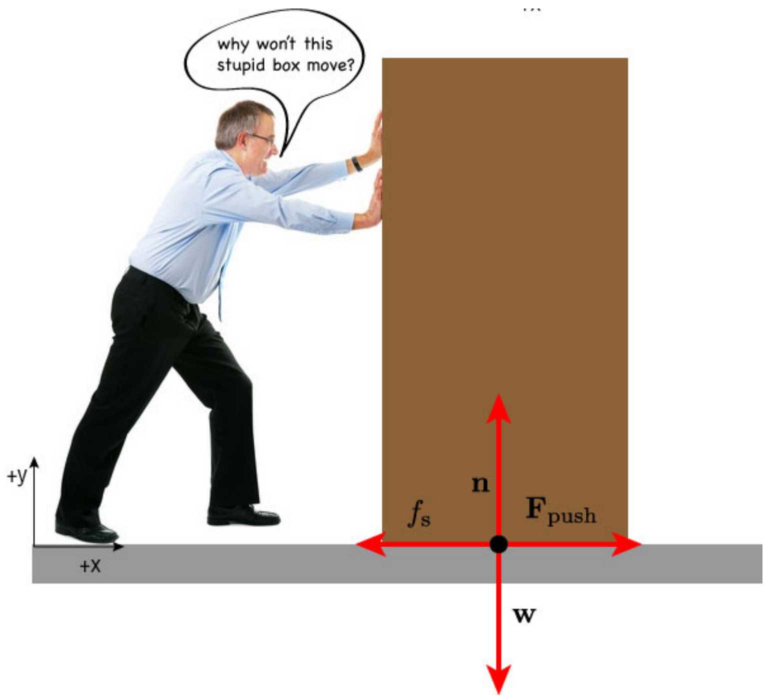
1. The direction of the static friction force is pointed opposite to the expected motion.
2. The magnitude of f_s changes so that the F_{net} is zero and the object stays in static equilibrium (i.e. not moving)
3. The magnitude of f_s cannot be larger than:

$$f_s^{\text{MAX}} = \mu_s n$$

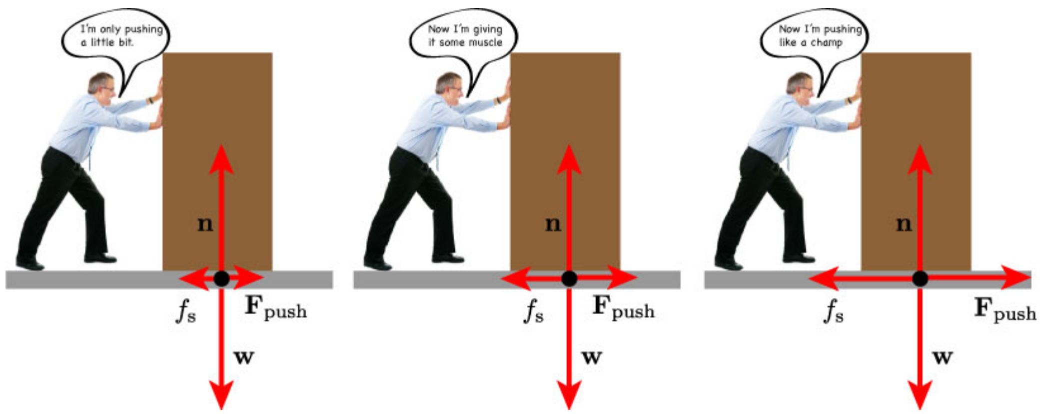
[μ_s : **coefficient of static friction** and n : **normal force**]

1) The direction of the static friction force f_s is pointed opposite to the expected motion.

...



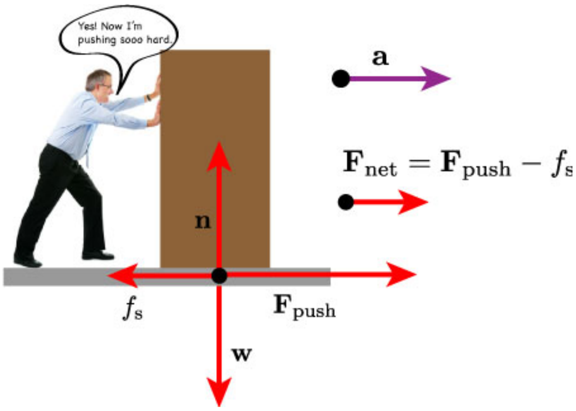
2) The magnitude of f_s changes so that the F_{net} is zero and the object stays in static equilibrium (i.e. not moving)



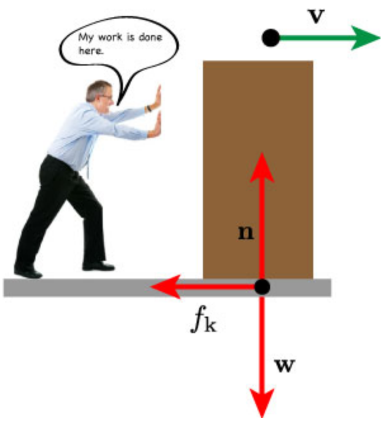
3) The magnitude of f_s cannot be larger than:

$$f_s^{\text{MAX}} = \mu_s n$$

Eventually, if the F_{push} is large enough, the μ_s will no longer be able to cancel the applied force and the box will begin to accelerate in the direction of F_{net}.



Which brings us to kinetic friction



Now that the box is in motion, the friction force is replaced by f_k .

$$f_k = \mu_k n$$

[μ_k : **coefficient of kinetic friction** and n : **normal force**]

Again, it's really a microscopic phenomenon, but we can make model about how it behaves. Kinetic friction is proportional to the normal force and doesn't depend on the velocity with which the object is sliding.

Some example coefficients of friction

Materials	**Static Friction** μ_s	**Kinetic Friction** μ_k
Glass on Glass	0.94	0.4
Ice on ice	0.1	0.02
Rubber on Dry Concrete	1.0	0.8
Rubber on Wet Concrete	0.7	0.5
Steel on Ice	0.1	0.05
Teflon on Teflon	0.04	0.04
Wood on Wood	0.35	0.3

Example Problem #18:

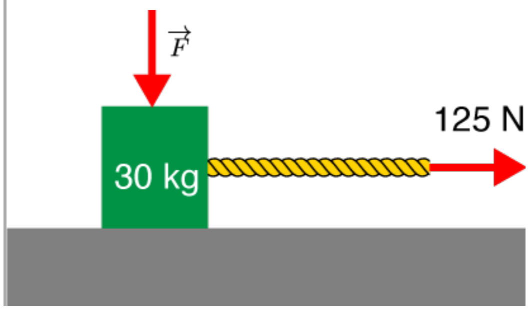
A 4000 kg rock is placed on a 15° slope. It stays put. What is the frictional force on the rock?

Example Problem #19:

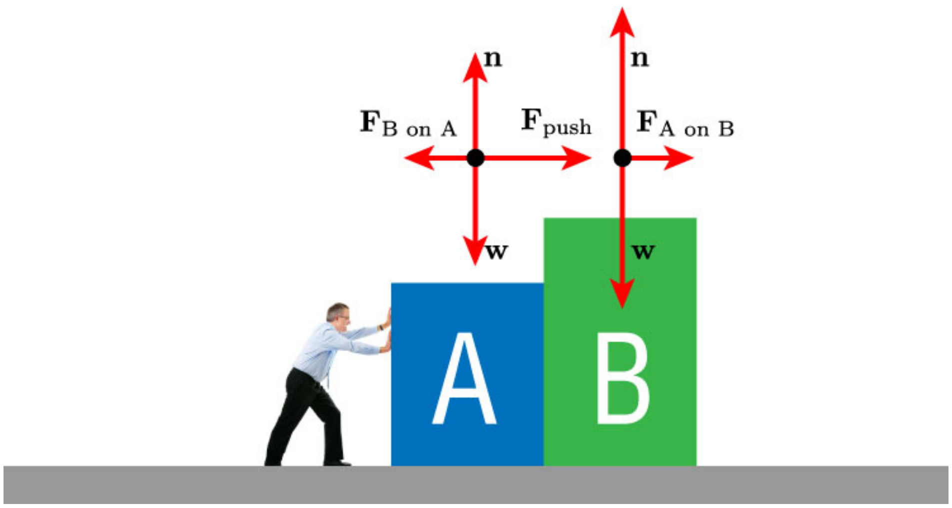
> I push a wooden block across a wood floor at a steady speed of 2.0 m/s. The block has a mass of 10kg. How much force am I exerting on the box? The μ_k for wood on wood is 0.20.

Example Problem #20:

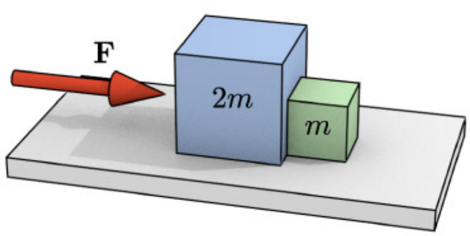
A downward force $\vec{F}$ is a applied to the box as shown. How strong does it need to be so that the box doesn't slide? The static friction coefficient is 0.35 between the box and the floor.



Interacting Objects

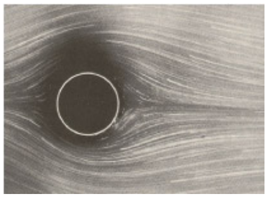


Example Problem #21:



A force of magnitude F pushes a block of mass 2m, which in turn pushes a block of mass m, as shown. The blocks are accelerated across a horizontal, frictionless surface. What is the magnitude (in terms of F) of the force that the smaller block exerts on the larger block?

Drag



The drag force D, is just as complicated as friction. Again, we have to make some approximations to reality in order to be able to work with drag. In general, drag will be:

1. Opposite to the direction of the velocity vector
2. Increases in magnitude as the objects speed increases.

We also have to assume:

1. the object is not very small or very big
2. the speed is less than a few hundred meters per second
3. the object is moving through the air near the earths surface

Drag

If the above conditions are met, then we can estimate the drag force by the following:

$$\mathrm{D} = \frac{1}{2} C_D \rho A v^2$$

where,

- C_D is the drag coefficient
- ρ is the density of air
- A is the cross-sectional area of the object
- v is the speed of the object.

Terminal Velocity

We know the F_{net} is proportional to the acceleration of an object. In a vacuum, for an object in free fall, the only force which contributes to F_{net} is the force of gravity: F_G.

However, in the presence of a medium (air), then D, the drag force must also be accounted for in the calculation of F_{net}.

$$F_{\text{net}} = D - F_G$$

Terminal Velocity

Eventually, since Drag is proportional to v^2, the two forces will cancel, leading to an $F_{\text{net}} = 0$, and thus no acceleration. This is terminal velocity.

$$D = F_G$$

$$\frac{1}{2} C \rho A v_t^2 - F_G = 0$$

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}$$

Forces, in Modern Physics

Property/Interaction	Gravitation	Weak	Electromagnetic	Strong	
			(Electroweak)	Fundamental	Residual
Acts on:	Mass - Energy	Flavor	Electric charge	Color charge	Atomic nuclei
Particles experiencing:	All	Quarks, leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Not yet observed (Graviton hypothesised)	$W^+ W^- Z^0$	γ (photon)	Gluons	Mesons

We've learned a lot since Newton wrote the 3 laws. The current understanding of interactions in nature is described by considering Four (or three) fundamental interactions.

- 1. Gravitation: This interaction acts on all objects. It's a force that acts between any two things and causes them to move closer. We see its manifestation in the orbits of planets as well as the perpetual attraction between our bodies and the ground.
- 2. & 3. Electroweak: The electroweak interaction is broken down into two other interactions: the *weak* and the *electromagnetic.* The weak force is responsible for radioactive decay of subatomic particles. The electromagnetic force manifests as either attractions or repulsions between electrically charges materials and particles.
- 4. Strong: This interaction occurs between sub-nuclear species such as protons and neutrons. It binds them together at very small distances. This interaction is responsible for keeping ordinary matter stable.