Work and Energy

Introduction

This is word that means a lot of things depending on the context:

1. Energy Consumption of a Household
2. Energy Drinks
3. Auras & Spiritual Energy
4. Renewable Energy

Energy is another example of a word that we use very often in regular speech. However, in physics it has a specific meaning.

Energy: basics

James Prescott Joule 1818-1889

Energy is a scalar quantity.

It has SI units of: $\mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}^2$

The unit is called a Joule, after Mr. Joule

1 Joule = 1 Newton × 1 meter

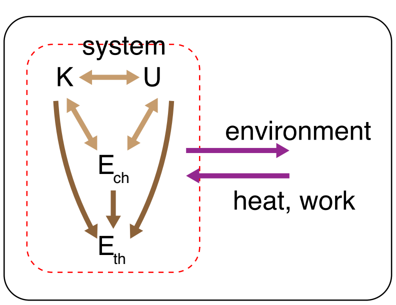
The different energies of a system can be transferred between each other.

$$E_{ch} \longrightarrow K$$

$$K \longrightarrow E_{th}$$

$$U \longrightarrow K$$

Work



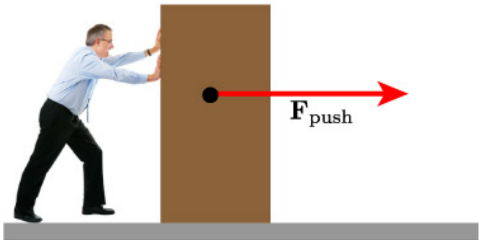
Again, we might have word problems here.

For us, Work will be defined as the process of transferring energy between a well-defined system and the environment.

The energy may go from the system to the environment. Or, it may go from the environment to the system

An example of work

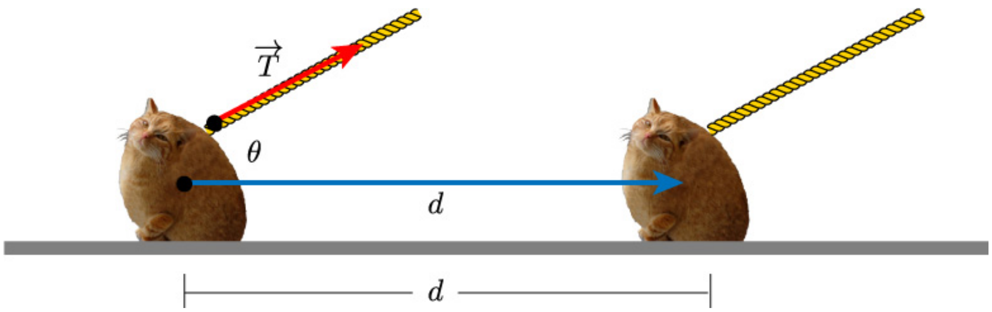
A constant force is applied this box as it moves across the floor.



The work done is equal given by: $W = Fd$.

This is for the case of a constant force which is applied in the same direction as the box is moving.

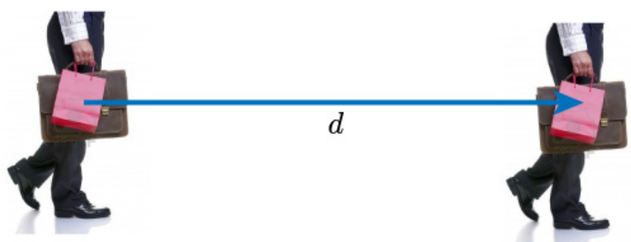
But what if the force is not in the exact same direction.



Now we have to consider the component of the force which does point in the direction of the distance traveled.

$$W = F_{\|} d = \mathrm{F} \cdot \mathrm{d} = Fd\cos\theta$$

What is the work being done on the bag by the person carrying it?



Kinetic Energy

We said there we many different forms energy can take (chemical, thermal, potential, etc)

Kinetic Energy is the energy associated with moving objects.

$$\boxed{KE = \frac{1}{2}mv^2}$$

Some example kinetic energies

1. Ant walking: 1×10^{-8} J
2. Person walking: 1×10^{-5} J
3. Bullet: 5000 J
4. Car @ 100 kph ~ 5×10^{5} J

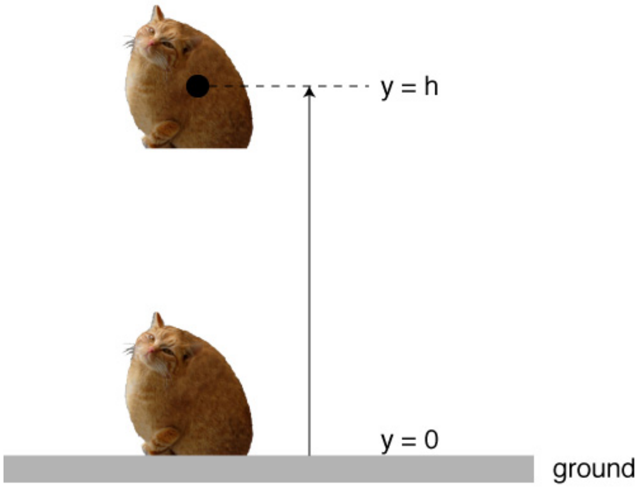
5. Fast train ~ 1×10^{10} J

Work and Kinetic Energy

Since the work is related to the amount of energy either entering or leaving a system, we can establish a relationship between the work done on an object and the kinetic energy.

$$W = KE_f - KE_0 \; = \frac{1}{2}mv_{\mathrm{f}}^2 - \frac{1}{2}mv_0^2$$

Potential Energy



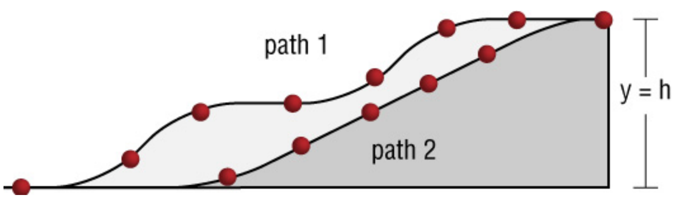
We can store energy in a gravitational system by separating the two objects.

The cat, when lifted off the ground, has a gravitational potential energy.

$$\boxed{U_g = mgh}$$

Potential Energy is only dependent on the height!

It doesn't matter the path an object takes while climbing higher away from the earth. Any route which ends at the same point will impart the same amount of gravitational potential energy to the object.



Work and Potential Energy

Changing the potential energy of an object requires work.

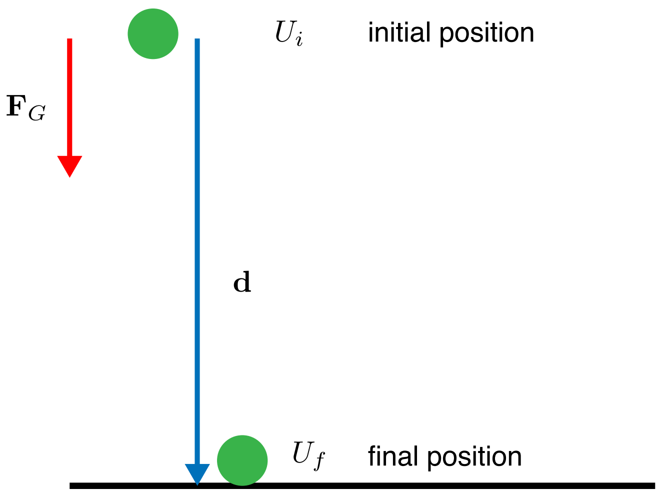
$$\Delta U = -W$$

In the case of gravity:

"The change in the gravitational potential energy of an object is equal to the negative of the work done by the gravitational force"

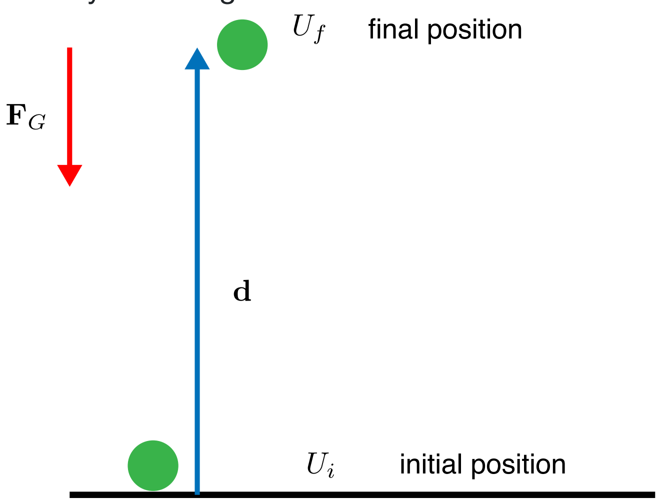
$$\Delta U_G = -W_G$$

Gravity does positive work



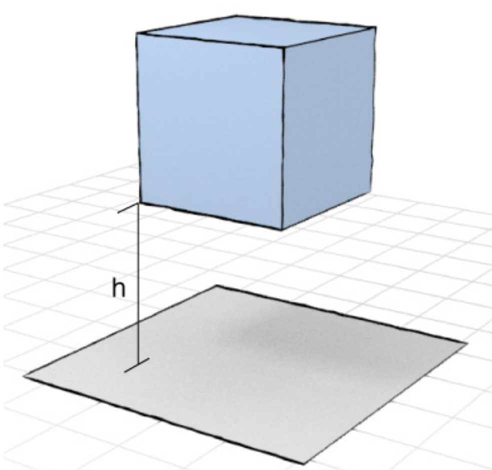
In this case we let an object drop, and gravity does positive work.

Gravity does negative work



In this case, we raise an object, and gravity does negative work.

Work - Energy



Let's now take a look at systems which have both kinetic and potential energy.

For example: this box which is located at a distance h above the ground.

Usually, we'll call the ground $h = 0$.

If these are the only two types of energy in the system, then we have a very special situation.

We can define the total *mechanical energy* energy of the system as the kinetic + potential.

$$E_{\text{mechanical}} = KE + U$$

Conservation of Mechanical Energy

If we don't worry about any of those retarding forces like friction or air resistance, then we can say that the total mechanical energy of a system remains the same as time advances.

$$E_i^{\text{mech}} = E_f^{\text{mech}}$$

or

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$$

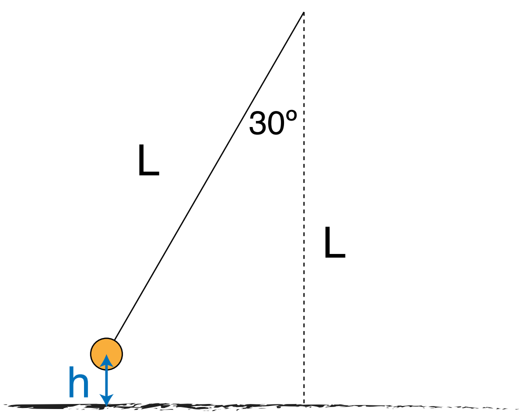
Fig. 1

Upon dropping a ball, the total mechanical energy is **conserved**.

$$E_{\text{mech}} = \text{constant} = KE + U_{\text{grav}}$$

Knowing this conservation law, we can use either h to find v, or v to find h.

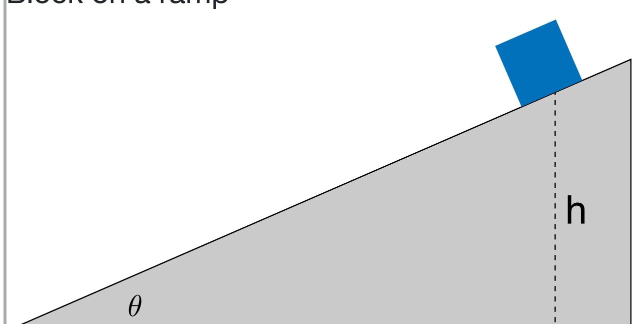
A car on a hill.



If L = 4 meters, how high is the ball? That is, find the distance h.

Example Problem #1:

Block on a ramp



Here's a 5 kg block on a ramp, where $\theta = 25^\circ$, and h = 20 meters. Find the work done by gravity and the final speed.

We want to calculate the work done by gravity:

$$W = \mathbf{F}_G \cdot \mathbf{d}$$

So, we need to decompose the two vectors into components. Let's call the horizontal direction +x to the left, and the vertical +y up. Thus, in unit vector notion, the force of gravity will be:

$$\mathrm{F}_G = -mg\hat{\mathrm{j}}$$

and the displacement will be:

$$\mathrm{d} = \frac{h}{\tan\theta}\hat{\mathrm{i}} - h\hat{\mathrm{j}}$$

Now, we can execute the dot product using unit vectors:

$$W = \mathrm{F}_G \cdot \mathrm{d} = \left[-mg\hat{\mathrm{j}}\right] \cdot \left[\frac{h}{\tan\theta}\hat{\mathrm{i}} - h\hat{\mathrm{j}}\right]$$

Since $\hat{\mathrm{i}} \cdot \hat{\mathrm{i}} = 1$ and $\hat{\mathrm{i}} \cdot \hat{\mathrm{j}} = 0$ this dot product simplifies to:

$$W = mgh$$

which will be the work done by gravity. Notice how that is same as if we just let the box fall from a distance h above the ground. This situation occurs because the force of gravity is a *conservative force*, i.e. it is path independent.

To find the speed at the bottom of the ramp, we can use the work-energy theorem which relates the work done to the change in kinetic energy:

$$\Delta KE = \text{Work done}$$

Initially, the box is at rest, so there is no kinetic energy:

$$KE_i = 0$$

The kinetic energy at the bottom of the ramp will be given by:

$$KE_f = \frac{1}{2}mv^2$$

Thus, the ΔKE will be $\frac{1}{2}mv^2$ Setting this equal to the work done that we calculated above:

$$mgh = \frac{1}{2}mv^2$$

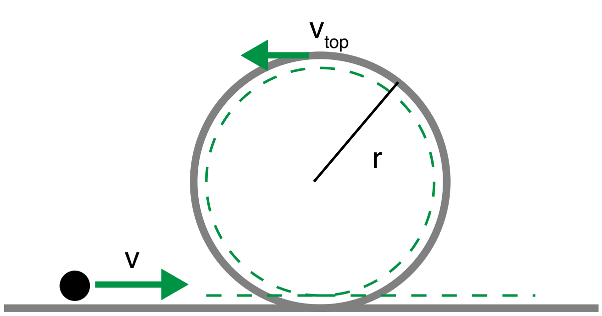
allows us to solve for v:

$$v = \sqrt{2gh}$$

This result should look familiar.

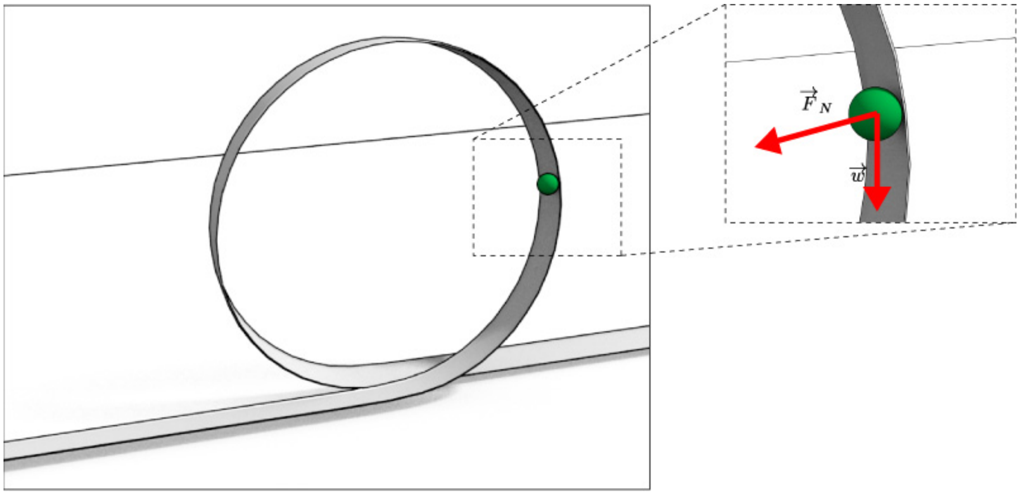
Notice: we use regular, non-rotated coordinates for this problem. Try it with rotated axes and see what you get. Should it be different?

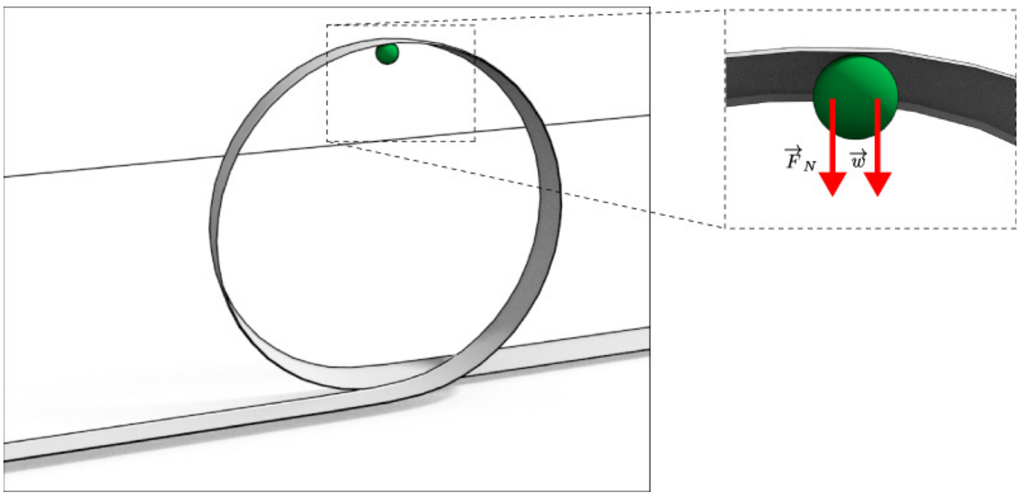
Ex: The Loop the Loop



We saw that if the velocity of the object was large enough, then it would remain inside in contact with the loop.

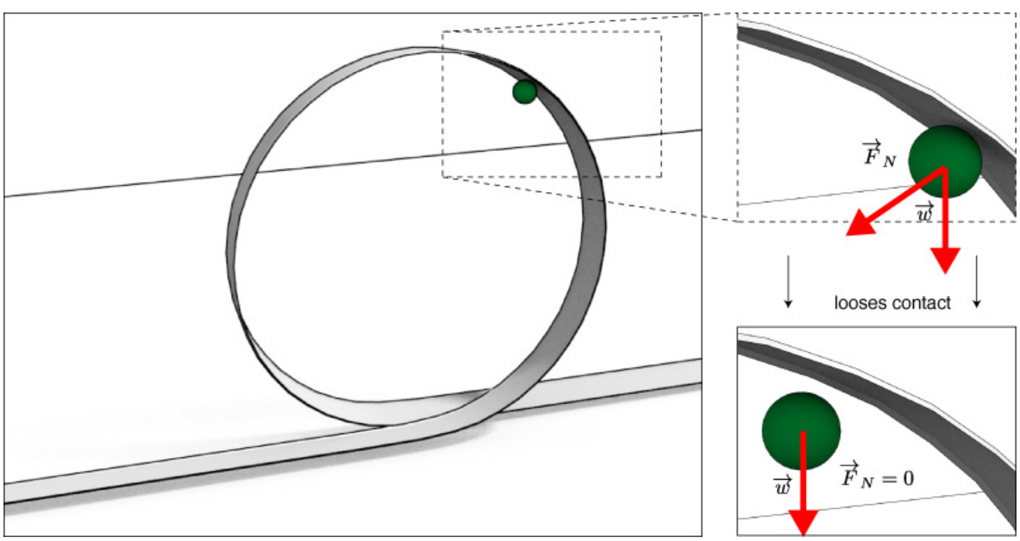
A successful loop-the-loop looks like this. The ball remains in physical contact with the road at all times. (This is another way of saying the normal force is *not* zero.)

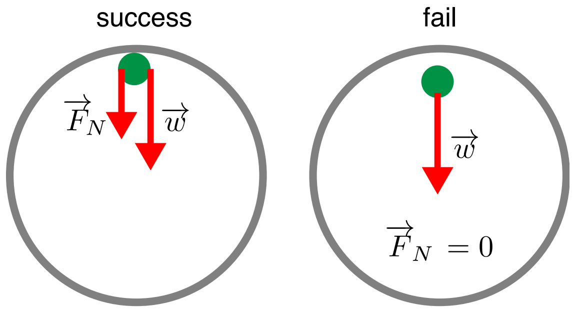




An unsuccessful loop-the-loop looks like this. The ball looses physical contact with the road at all times. (This is another way of saying the normal force *becomes* zero.)

In the case where the velocity is not fast enough to get the ball all the way around the loop, the ball looses contact with the road near the top.





At the top of the loop, the sum of forces pointing towards the ground will be given by:

$$\sum F_{\text{ground}} = w + F_{\text{N}} = ma$$

Since this is circular motion, we have

$$ma = m\frac{v^2}{r}$$

Thus we can write:

$$\underbrace{mg}_{\text{weight}} + F_N = m\frac{v^2}{r}$$

Conservative and Non-conservative Forces

Most of the forces we have seen can be classified as *non-conservative.*

The exception is Gravity.

Here are two definitions for a conservative force:

1. A force is *conservative* if the work it does to move an object from point A to point B does not depend on the route chosen (aka the path) to get from point A to point B.

2. A force is *conservative* if there is zero net work done while moving an object around a closed loop, that is, around a path that has the same point for the beginning and the end.

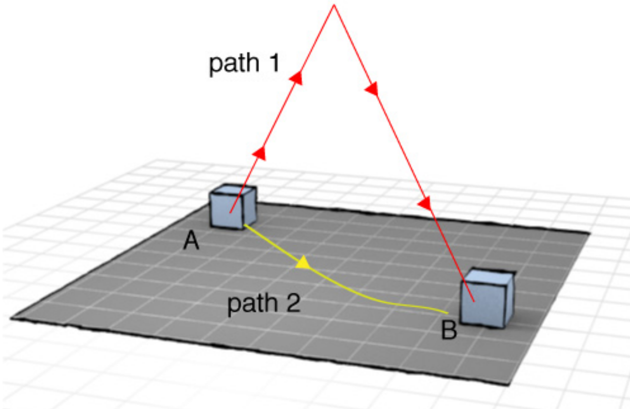
Examples of Conservative Forces

- Gravitational force
- Elastic spring force
- Electric force

and Non-conservative forces:

- Static and kinetic frictional forces
- Air resistance
- Tension
- Normal force

Shown are two paths that a box can take to get from point A to point B

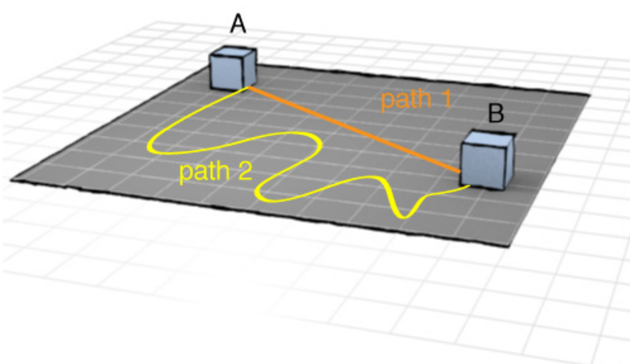


If we look at the change in potential energy, ΔU_{grav}, then we'll see that the Work done by gravity on the box in either case is equal to zero. And, for any other path you can imagine, it's also zero.

$$W = -\Delta U_G = mg(h - h) = 0$$

Here is an example of a conservative force interacting with a box. If we slide the box from point A to B along path 1, we can calculate the work done by figuring out the work done by gravity to go from the ground up to the apex, and then back down to the ground. Adding these two works together will result in zero, since on the way up, gravity is pointing against the displacement vector: [$F_G d\cos(180°)$] and on the way down, gravity will be in the same direction as the displacement vector: [$F_G d\cos(0°)$]. Thus, these two values are equal in magnitude but opposite in sign and so when that are added together will be equal to zero. In the case of path 2, the work done by gravity will be zero also, since there is no change in height along the path. Thus, no matter how we chose to go from the A to B, the net work done by gravity will be zero. This is the definition of a conservative force.

Moving a box along 2 paths in the presence of friction.



The situation is very different if we ask about the work done against friction during these two paths. Path 2 will require more work than path 1.

Now, if you want to move the box from A to B, but are asking about the work done by friction on the box during the motion, the shorter path (path 1) will result in less work. Since the force of friction is always opposite to the direction of motion, there will never be the case of canceling out like we had in the gravitational case. Path 2 will lead to more work being done by kinetic friction, which is therefore considered a *non-conservative* force.

And so, forces like friction and air resistance are considered non-conservative.

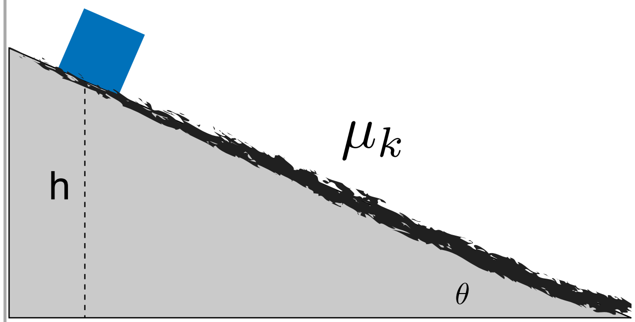
We can however account for these forces in our analysis of certain systems.

$$W_{\text{NC}} = E_f - E_0$$

If the change in energy is non-zero, bewteen the initial and final times of a mechanical system, then there must be some non-conservative forces doing work during the motion.

Example Problem #2:

Now, we have a ramp that has some friction.



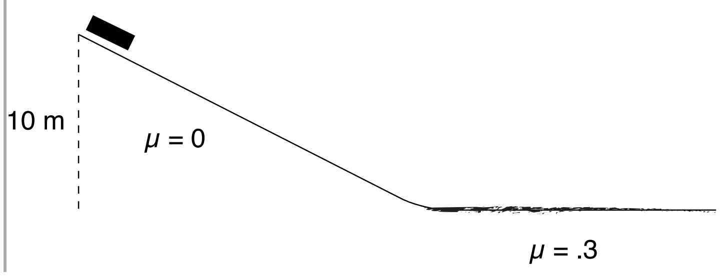
How can we describe the motion of a box on this ramp?

Example Problem #3:

A child, starting from rest, slides down a slide. On the way down, a kinetic frictional force (a nonconservative force) acts on her. The child has a mass of 50.0 kg, and the height of the slide is 18 m. If the kinetic frictional force does 6.50×10^3 Joules of work, how fast is the child going at the bottom of the slide?

Example Problem #4:

If the object slides down a hill with an elevation of 10 m, where will it stop? Consider the sloped part frictionless and the rough part at the bottom having a μ_k equal to 0.3.



Power

Again, here's another word we have to be very careful with. We might be used to calling anything that seems strong 'more powerful'.

For us however, we'll need to restrict our use of the word power to describe how fast a process converts energy.

$$\text{Power} = P = \frac{\text{change in energy}}{\text{time}}$$

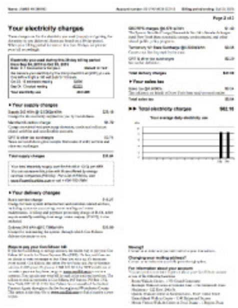
The SI unit of power is called the watt:

$$1\text{watt} = 1\text{joule/second} = 1\text{kg} \cdot \text{m}^2/\text{s}^3$$

A unit of power in the US Customary system is horsepower:
1 hp = 746 W

Units of [power - time] can also be used to express energy (e.g. kilowatt-hour)

$$1\text{kWh} = (1000\text{W})(3600\text{s}) = 3.6 \times 10^6\text{J}$$



Electricity you used during this 29 day billing period from Sep 04, 2013 to Oct 03, 2013
Rate: EL1 Residential or Religious

We measure your electricity by how many kilowatt hours (kWh) you use. One kWh will light a 100 watt bulb for 10 hours.

Oct 03, 13 estimated reading	52566
Sep 04, 13 actual reading	-52323
Your electricity use	**243 kWh**

We should be able to understand these mysterious documents now.

How many Joules of energy did I use?

How many pushups would I have to do to generate that much energy?

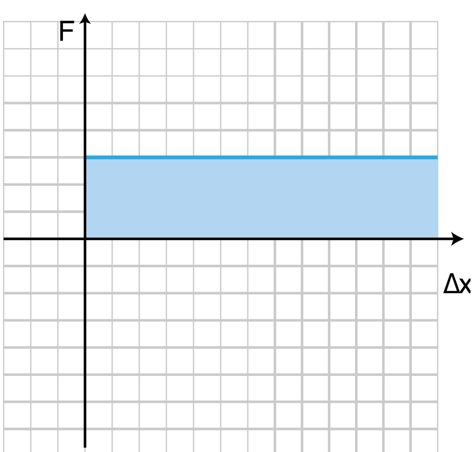
Example Problem #5:

I used to work out on a rowing machine. Each time I pulled on the rowing bar (which simulates the oars), it moved a distance of 1.1 m in a time of 1.6 s. There was a little display that showed my power and it said 90 Watts. How large was the force that I applied to the rowing bar?

Example Problem #6:

A go kart weighs 1000 kg and accelerates at 4 m/s^2 for 4 seconds. What is the average power?

Variable and Constant Forces

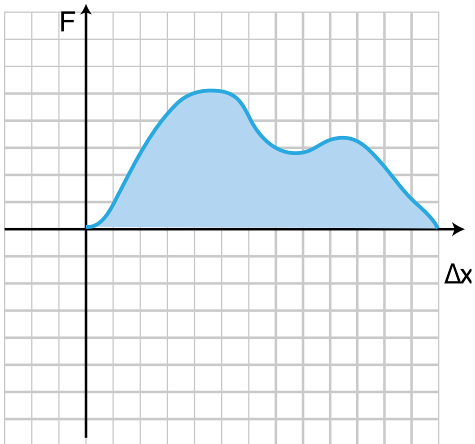


In the most basic situation, the force applied to an object was constant. When this is the case, the Work was given by:

$$W = \mathrm{F} \cdot \mathrm{d}$$

This is also equal to the "area under the curve"

Work done by a *variable* force

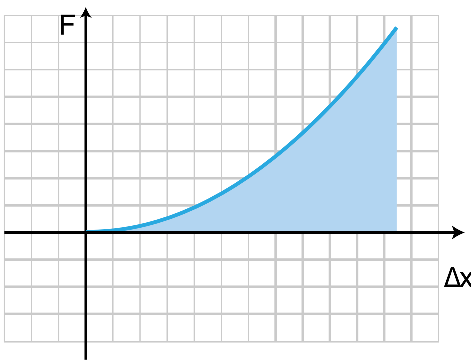


However, it's also possible that the work will vary as a function of distance. In this case, the work will not be given by the standard form, because, the F is not constant.

$$W \neq \mathrm{F} \cdot \mathrm{d}$$

However, the work is still equal to the "area under the curve"

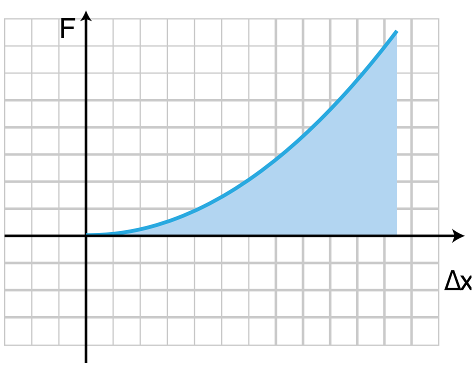
Work done by a *variable* force



We don't have the tools (i.e. integration) to be able to do much with this at this level of physics.

Work done by a *variable* force

To deal with these situations, we'll have to use some integration techniques.



Here's an applied force that varies as the square of the distance.

We could figure out the area under the curve, which does equal the work, but performing the integral

$$W = \int_{x_i}^{x_f} F(x) dx$$

In this case:

$$W=\int_{x_i}^{x_f} x^2\,dx=\frac{1}{3}x^3\Big|_{x_i}^{x_f}$$

In the case of 3d:

If our forces are variable and 3 dimensional, (and perhaps not pointed directly in the direction of the displacement), then we have to use some more developed vector calc:

$$dW=\mathrm{F}\cdot\mathrm{dr}=F_x dx+F_y dy+F_z dz$$

Or, if we integrate over the distance:

$$W=\int_{r_i}^{r_f} dW=\int_{x_i}^{x_f} F_x dx+\int_{y_i}^{y_f} F_y dy+\int_{z_i}^{z_f} F_z dz$$

Since work is a scalar, then these terms will just add up without any complications.

Recover the work-kinetic energy equation?

$$W=\int_{x_i}^{x_f} F(x)dx$$

We should be able to show that this gets back to kinetic energy somehow...

Based on the above:

$$\Delta U(x)=-W=-F\Delta x$$

Now we can take the derivative with respect to x:

$$\boxed{F(x)=-\frac{dU(x)}{dx}}$$

Example Problem #7:

Draw plots of the gravitational potential energy and the gravitational force for an object as a function of height above the earth. Consider only small distances above the surface of the earth.

Example Problem #8:

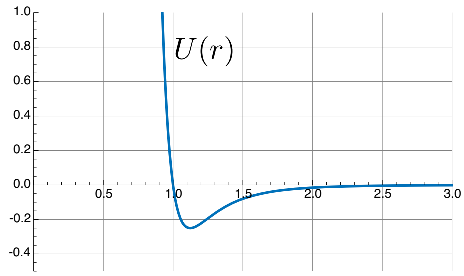
If the force from an elastic band when stretched is given by$F=-kx$, what would the change in potential energy be for an object attached to the rubber band and pulled back by a distance d? (k is a constant that is determined by the rubber band material) Plot the force and $U_{\text{rubber band}}$ as functions of distance.

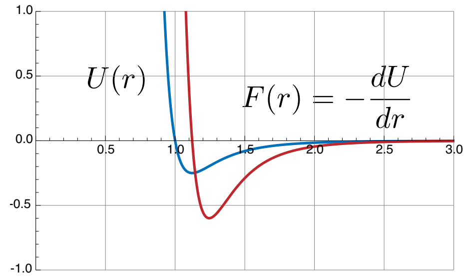
Example Problem #9:

A diatomic molecule has two atoms (H_2 for example). If the potential energy of such a system is given by:

$$U=\frac{A}{r^{12}}-\frac{B}{r^6}$$

Find the equilibrium separation of such a molecule. That is, find a distance$r$ that will lead to zero net force on each atom.(A and B are just constants)





Here are plots of the Potential function, and its derivative (i.e. force).

$$U = \frac{1}{r^{12}} - \frac{1}{r^6}$$

Force Microscopy

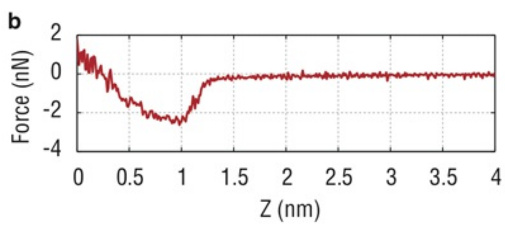


Figure 4.8: **a**: Small amplitude (9 Å) approach curve and **b**: the results of numerical integration to obtain a force distance curve, $T = 4$ K

More info: [Intro to AFM]

Emmy

23 March 1882 – 14 April 1935

Emmy Noether was a theoretical physicist who did pioneering work in the study of conservation principles.