Torque

1. Introduction

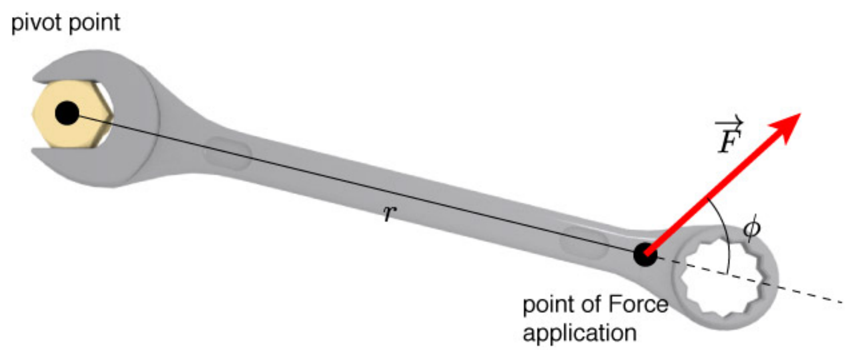
Torque will be used to quantify forces applied to bodies which can rotate.

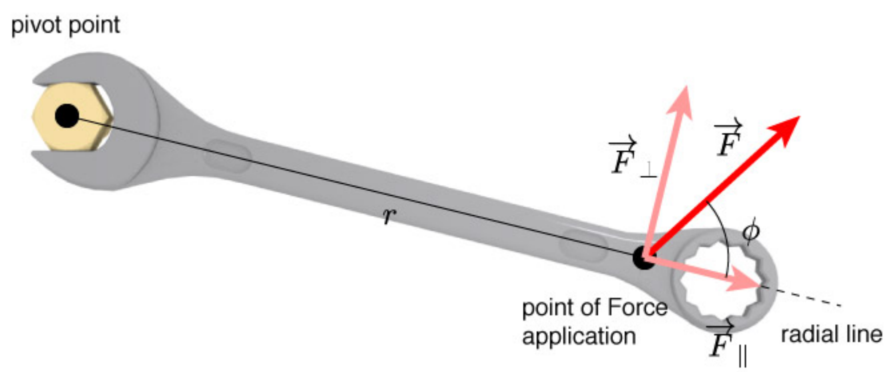
Previously, in the particle model, it didn't matter where on a body we applied a force. However, with *rigid bodies*, that is no longer true.

(Things are no longer point particles.)

2. Torque

1. The magnitude of the force
2. The distance r, from the pivot point to where the force is applied
3. The angle at which the force is applied





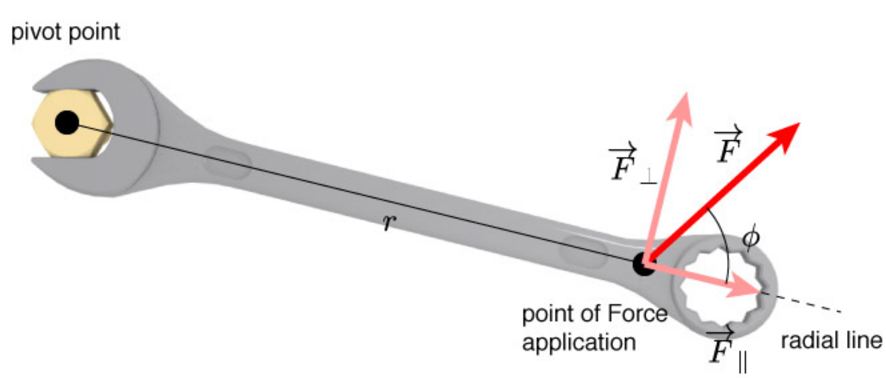
$F_\perp$ contributes to rotating the wrench

$F_\parallel$ does not contribute to the rotation at all.

We know from experience that

1. it's easier to open a door when you push at the point that's farthest from the hinges
2. you have to push in the direction you want it to go.

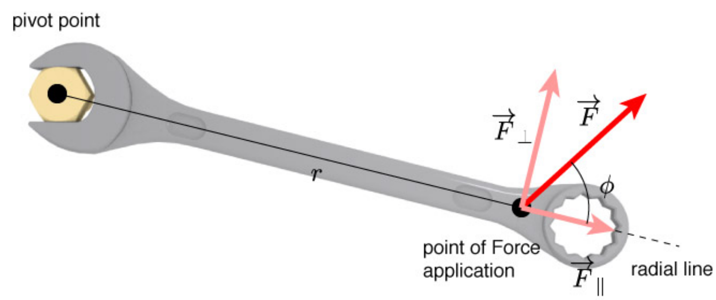
(let's turn that into math)



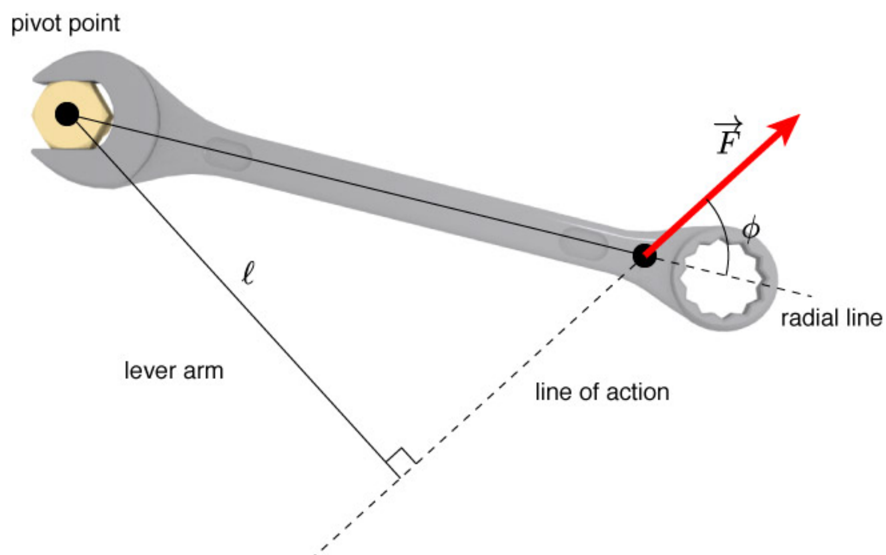
Torque, τ will be given by:

$$\tau = rF_\perp$$

Quick Question 1

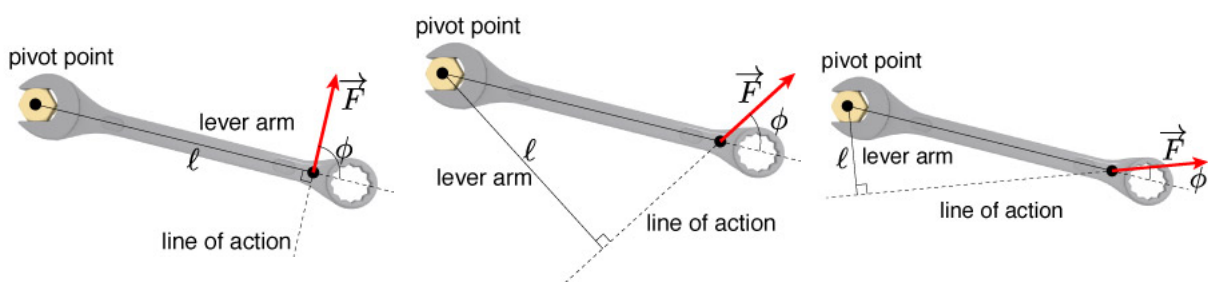


Find $F_\perp$ based on $\vec{F}$ and ϕ



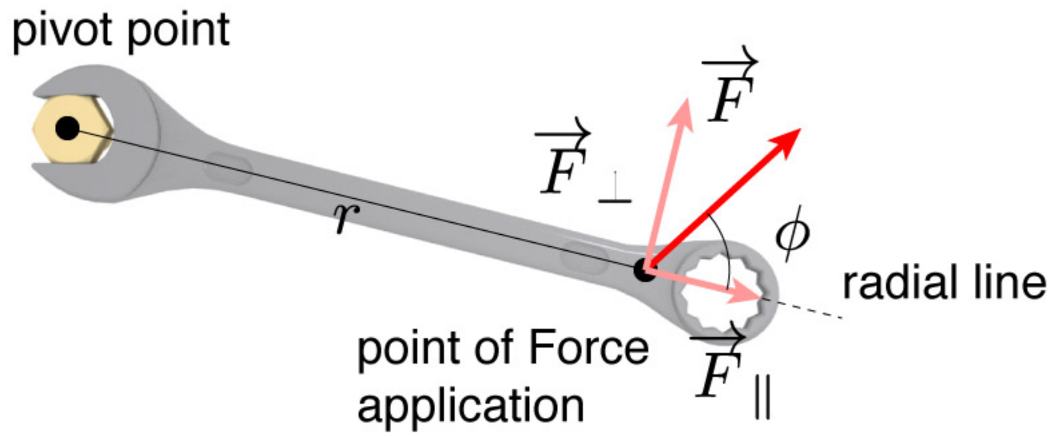
2.1 Lever arm changes

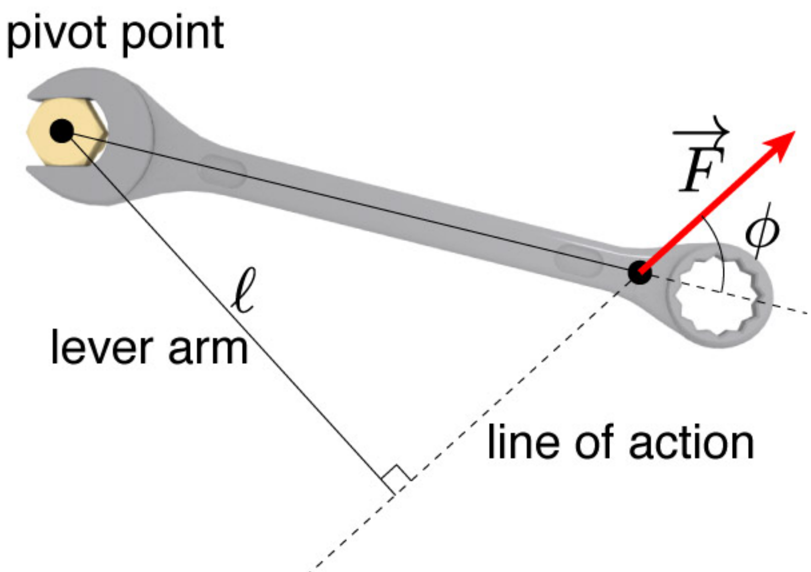
As the angle of the line of action changes, then the length of the 'lever arm' changes.



$$\tau = F\ell$$

In general...





Either way you set it up, the torque is still given by the force applied, times the distance away from the pivot point, and the sine of the angle between the force and the radius of motion.

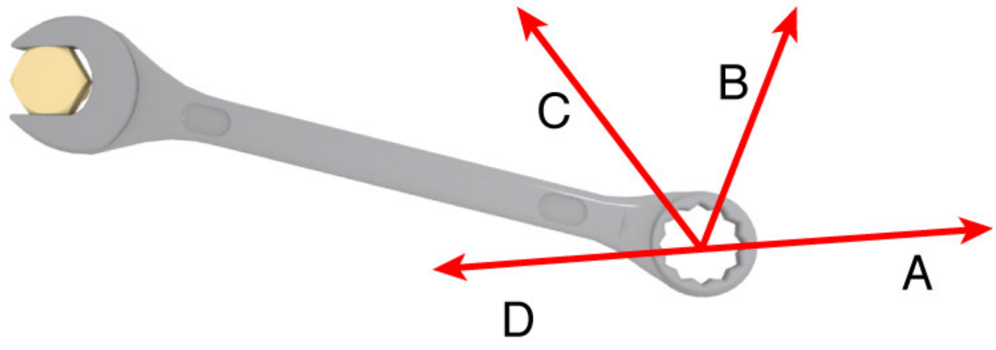
$$\tau = rF_\perp = rF\sin\phi$$

or,

$$\tau = F\ell = Fr\sin\phi$$

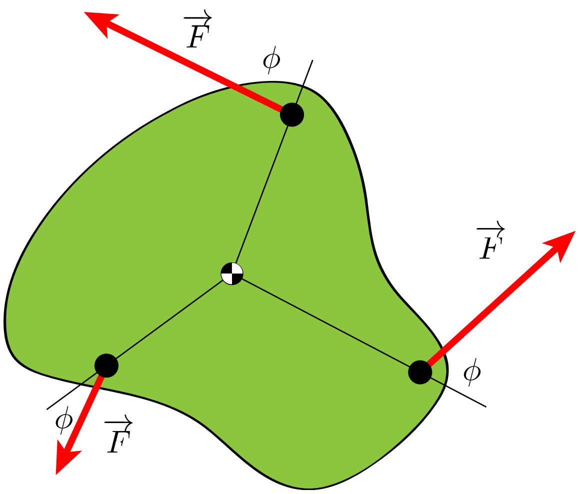
Quick Question 2

Which force will provide the largest magnitude of torque on the wrench? (E if torque is the same for each.)

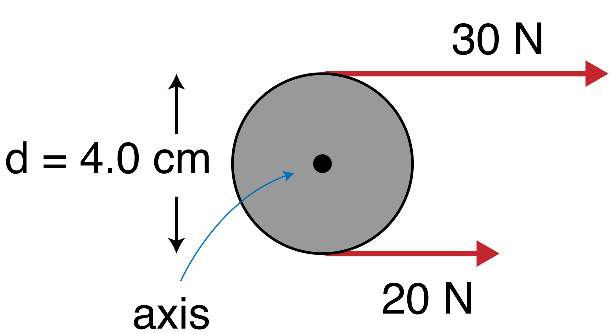


3. Net Torques

Just like the net force, we can add up all the different torques applied to an object to see if, and which way, it will rotate.



$$\sum \tau = \tau_1 + \tau_2 + \tau_3 \ldots$$

> Example Problem #1:
>
> 
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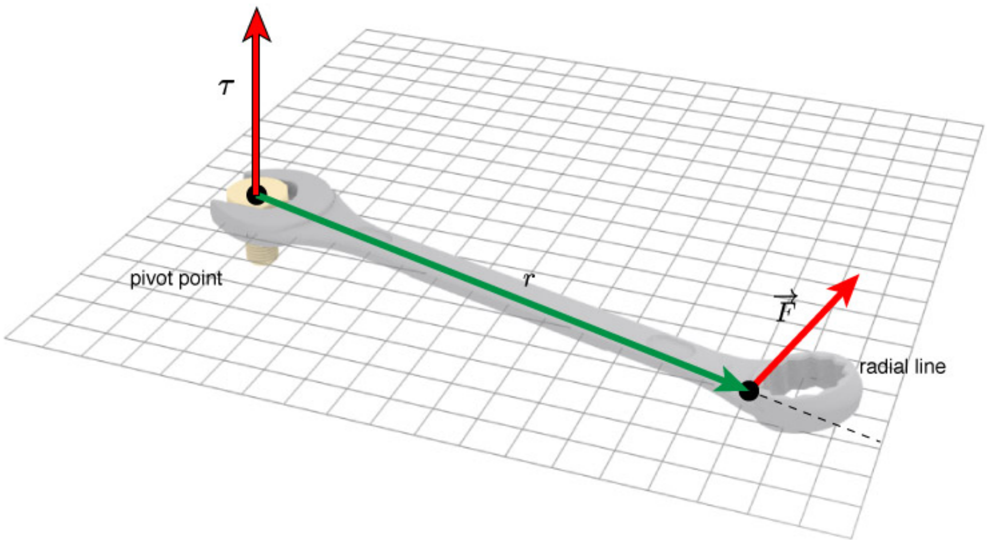
>
> What is the net torque around the labeled axis?

Torque

Torque, like force, is a vector

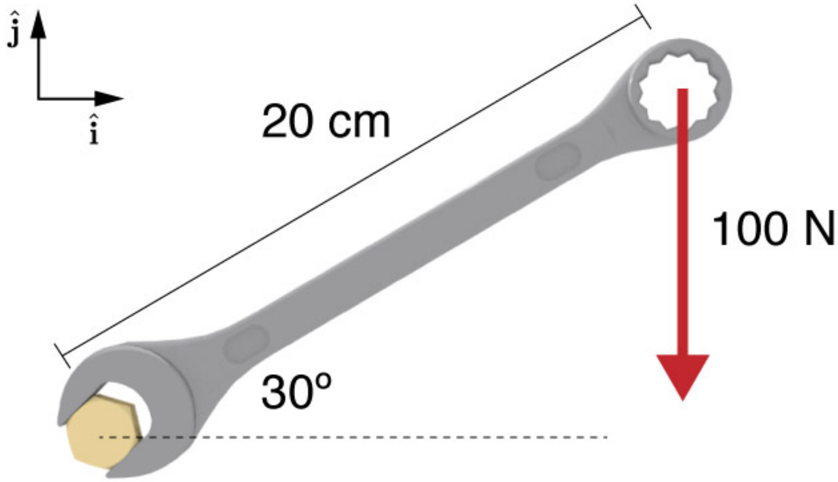


$$\boxed{\tau = |r||F|\sin\phi}$$

Quick Question 3

To open the door, which direction should the torque vector point?

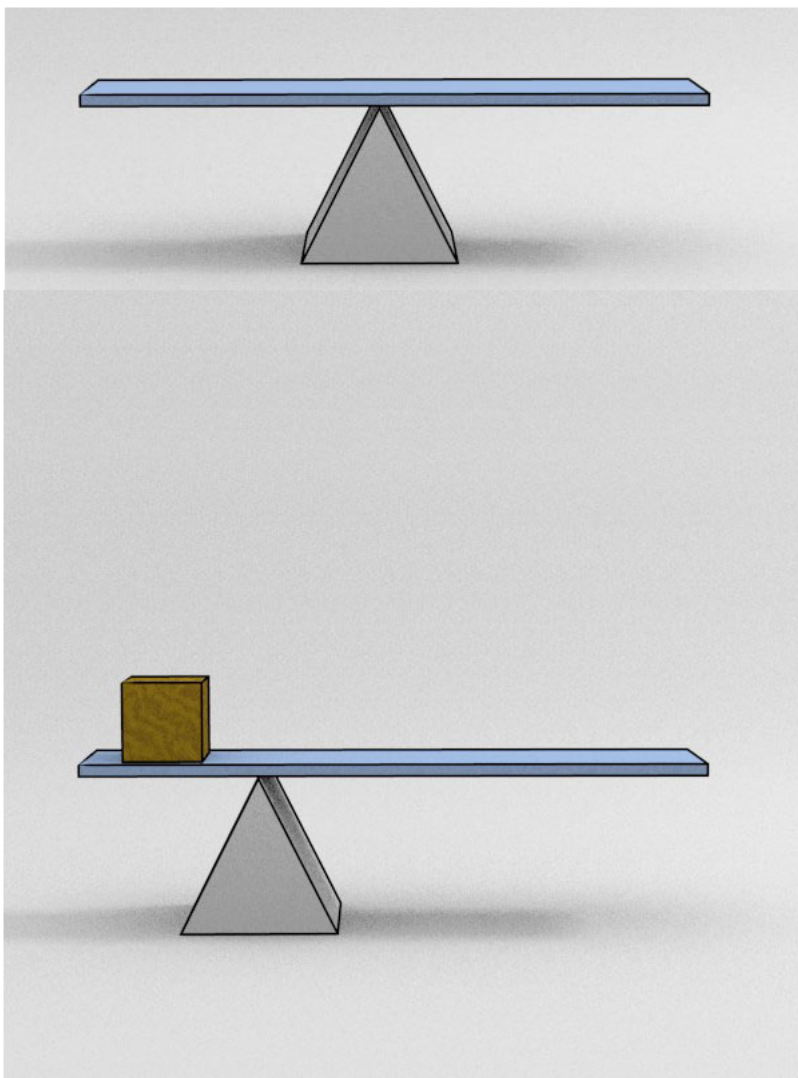
- a) $\mathbf{x}$
- b) $-\mathbf{x}$
- c) $\mathbf{y}$
- d) $-\mathbf{y}$
- e) $\mathbf{z}$
- f) $-\mathbf{z}$



What is the torque applied to the nut?

Fulcrums and Levers

Let's balance a board on a fulcrum.



Example Problem #2:

> **13.** A meter stick of mass 0.6 kg sits on a fulcrum located at the 0.3-m mark at equilibrium. At the 0.0-m mark hangs a mass *m*. What is *m*?
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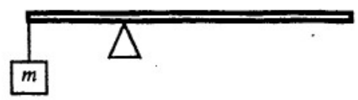
> 
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>
> A meter stick of mass 0.6 kg sits on a fulcrum located at the 30 cm mark at equilibrium. At the end of the stick (0.0 cm) hangs a mass m. What is m?

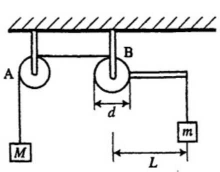
Quick Question 4

1. A force of some N is applied...? (3 parts)

2. What is the cross product of these two vectors...? (3 parts)

Quick Question 5

Pulley B hangs from the ceiling and has a diameter d. A string twined about the pulley leads around pulley A, hanging from the ceiling, and to a mass M. A beam of length L is attached to the pulley B itself and stretches out horizon- tally. A mass m is connected to the end. The system is in static equilibrium. (See figure.)



14. What is the magnitude of the torque due to the weight of mass *m* about the axis of pulley B?

A. *mgd/2*
B. *mgd*
C. *mgL/2*
D. *mgL*

15. Which gives an expression for *M*?

A. *mL/2d*
B. *mL/d*
C. *2mL/d*
D. *md/2L*

4. Moment of Rotational Inertia

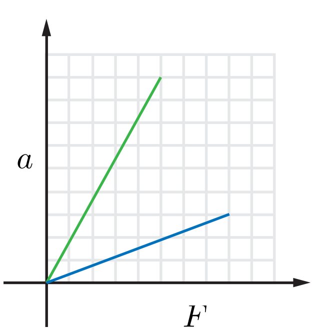
We saw before that a force caused a linear acceleration, with Newton's 2 ndlaw:

$$F = ma$$

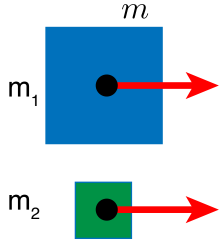
Since a torque is basically the rotational equivalent of force, then it should cause an angular acceleration.

$$\tau \rightarrow \alpha$$

However, we need a constant of proportionality between the two terms.



$$a = \frac{F}{m}$$



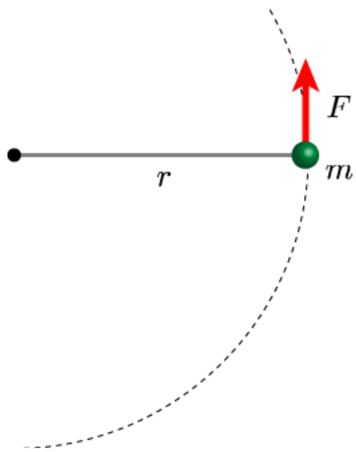
$$\boxed{\tau = I\alpha}$$

I is called the 'moment of inertia'.

It will describe how easy or hard it is to rotate a rigid body. (just like mass told us how hard or easy it was to move a body)

In the case of F, and linear motion, if we applied the same force to two different masses, then the heavier mass would have a smaller acceleration.

But, in the case of rotating objects, it's not just the mass that affects the angular rotation, but also how that mass is distributed.



In this case, the 2nd law for this particle with mass m, held a distance r from the center will be:

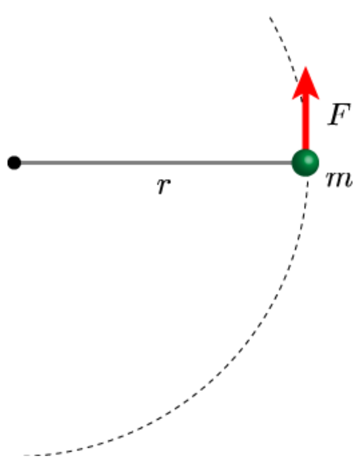
$$a_T = \frac{F}{m}$$

but the tangential acceleration is related to the angular acceleration by $a_T = \alpha r$. So we can write:

$$\alpha = \frac{F}{mr}$$

Using also $\tau = rF_\perp$, we can write this as:

$$\alpha = \frac{\tau}{mr^2}$$



Thus we have for a ball rotating around an axis:

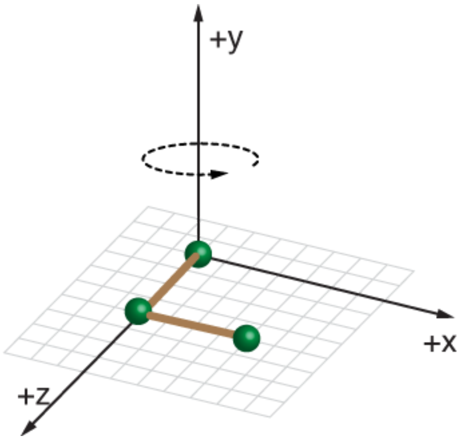
$$\alpha = \frac{\tau}{mr^2}$$

For this geometry, the moment of inertia, I, is given by:

$$I = mr^2$$

This makes sense, if the mass is heavier, then the angular acceleration will be less, and also if mass is farther away from the center, the angular acceleration will be less.

If we had multiple masses:



We can simply sum up all the moments of inertia for each little particle:

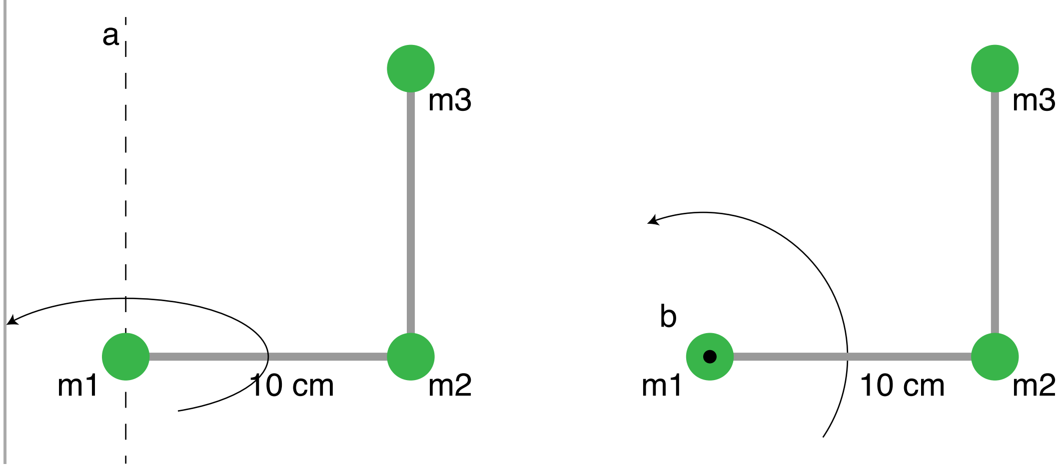
$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 \ldots = \sum mr^2$$

Example Problem #3:

Example

Calculate the moment of inertias for this shape based on the two axes shown.

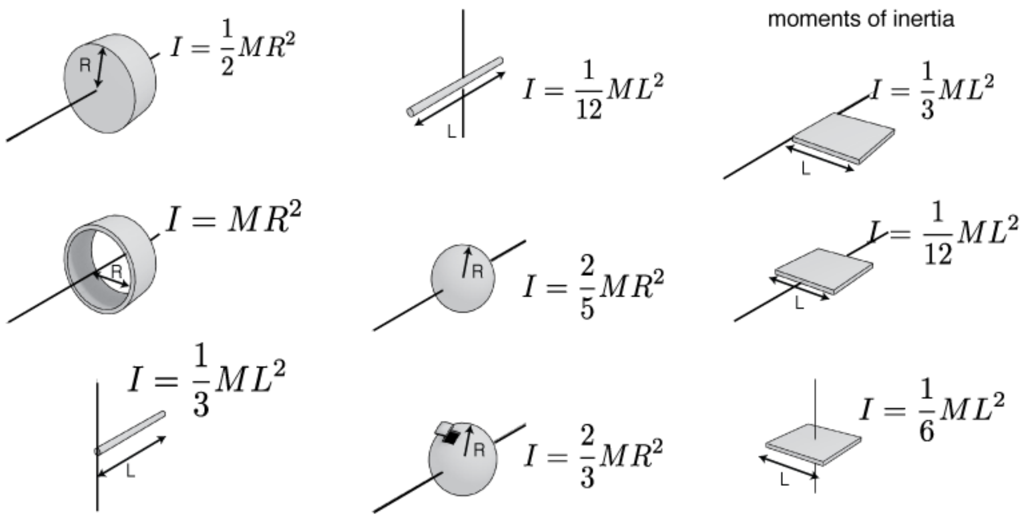
(m1 = 1.0 kg, m2 = 1.5 kg and m3 = 1.0 kg)



For shapes of arbitrary dimensions, things would get a little more complicated.

Even for a seemingly simple shape like a thin rod being rotated around one of its ends, we'd have to do some integral calculus to figure out the moment of inertia:

Moment of inertia for various geometries



> Example Problem #4:
>
> The engine in a plane can deliver 500 N•m to the propeller. The propeller has a mass of 40 kg and is 2.0 meters long (diameter). How long does it take to reach 2000 rpm?

> Example Problem #5:
>
> A telephone pole falls over in a storm. It is 7.0 meters tall and has a mass of 260 kg. Estimate the angular acceleration of the pole when it has fallen by $25°$ from the vertical.

Quick Question 6

a) What would the moment of inertia be for a regular wooden door...?

b) ...what will its angular acceleration be?

Energy of Rotation

The total mechanical energy of a system was the kinetic plus potential

$$E_{\text{mech}} = KE + U$$

However, KE needs to include both the translational energy: $\frac{1}{2}mv^2$ and the rotational energy:

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2$$

thus

$$E_{\text{mech}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh$$

Which object wins the race?

1. The Solid Cylinder
2. The Empty Hoop
3. It'll be a tie

Which can wins the race?

1. Chicken Broth
2. Cream 'O Chicken
3. It'll be a tie

5. Tale of Two Energies

This combination of motion demands that we account for both linear and rotational energy when describing a rolling object:

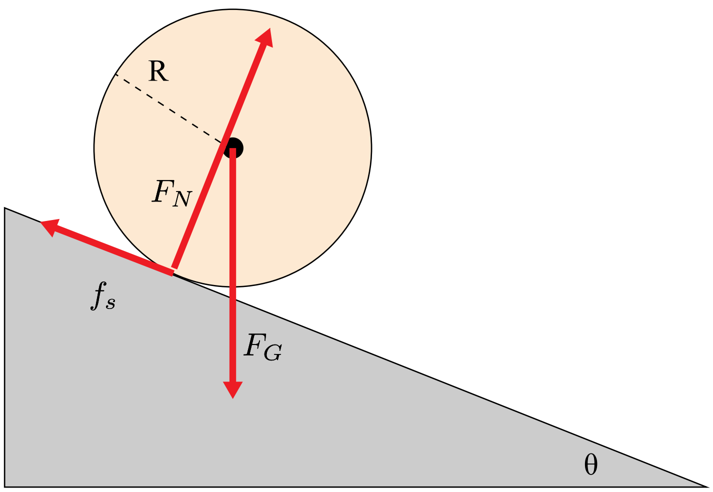
$$K_{\text{rolling w.o. slipping}} = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2$$

Just to be more specific

$$K_{\text{rolling w.o. slipping}} = \frac{1}{2}I_{\text{com}}\omega^2 + \frac{1}{2}Mv_{\text{com}}^2$$

We'll often need the geometrical constraint: $v = \omega r$ to deal with these two terms.

Quick Question 7



Which force creates a non-zero torque?

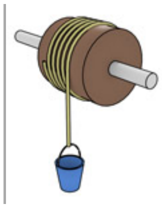
a) Static Friction
b) Gravity
c) Normal Force
d) They all create torques

Example Problem #6:

A spherical shell rolls down a ramp starting at height h. What is the speed of the sphere at the bottom?

Example Problem #7:

A bucket is attached to a string that is wrapped around a cylinder as shown. If the bucket is released from rest 1 meter above the ground, how long will it take to hit the floor?
(Mass of the bucket = 2.0 kg, mass of cylinder = 1 kg, radius of cylinder = 2.0 cm, mass of string = 0 kg)



6. Conservation of Angular Momentum

Magnitude

$$L = I\omega$$

Direction

$$\mathbf{L} = I\overrightarrow{\omega}$$

Conservation:

$$L_i = L_f$$

Angular momentum can be considered the *rotational analogue* of linear momentum. Its magnitude is given by the moment of rotational inertia times the angular velocity.

$$L = I\omega$$

Just like linear momentum, $\mathbf{p}$, angular momentum is also a vector: $\mathbf{L}$. Its direction will point in the same direction as the angular velocity vector. (i.e. perpendicular to the plane of rotation, and following the RHR.)

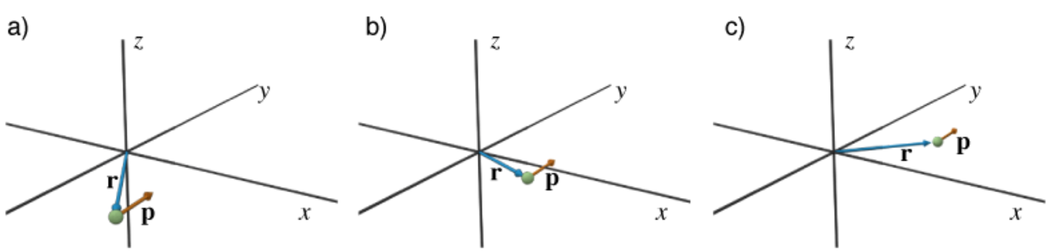
Another *conservation law:* In an isolated system, in the absence of external torques, the total angular momentum does not change.

$$L_i = L_f$$

Example Problem #8:

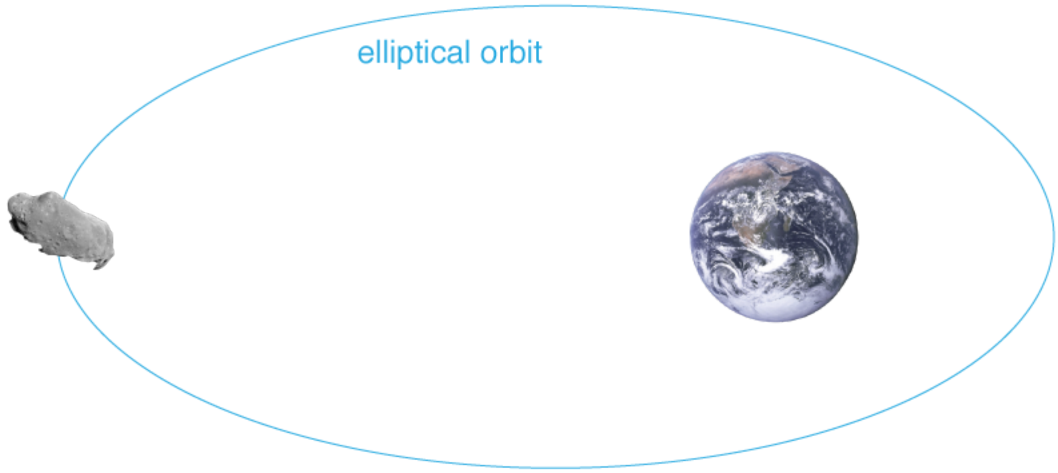
A kid (mass = 36 kg) stands at the center of a rotating disk (a.k.a merry-go-round). Its mass is 200 kg and rotates once every 2.5 seconds. If the kid walks 2.0 meters away from the center to reach the edge, what will the period of rotation be when he reaches the edge?

6.2 Angular Momentum - Particle



A particle is moving in the +y direction.

Angular momentum of a satellite



The 2nd Law

Earlier, we rephrased $\mathbf{F} = m\mathbf{a}$ as $\mathbf{F} = \frac{\Delta \mathbf{p}}{\Delta t}$

We can consider the rotational equivalent to this law:

$$\vec{\tau}_{\textbf{net}} = \frac{\Delta \mathbf{L}}{\Delta t}$$