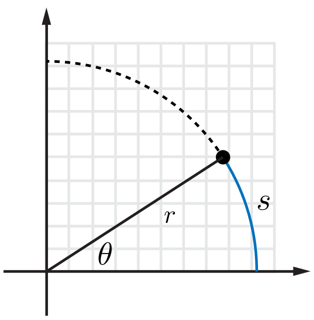
Rotational Kinematics

1. Introduction



The goal: to re-express our kinematics, dynamics, and forces in terms of rotating objects.

We'll have to use angular displacements instead of linear ones.

This chapter is all about how to describe motion of objects undergoing rotation. The next chapter will deal the understanding the causes of rotation. This is distinction is similar to that between kinematics and Newton's laws. Kinematics told us how object moved, Newton's laws told us why they moved.

Quick Question 1

What is the angular velocity of the record player?

Angular Acceleration

Of course, ω can change.

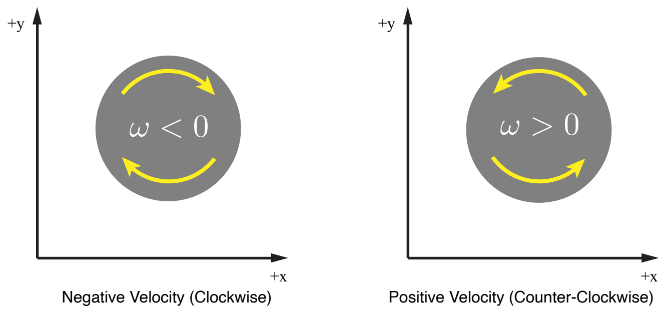
If that happens then we'll have to use an angular acceleration to describe that motion.

$$\overline{\alpha} = \frac{\Delta\omega}{\Delta t}$$

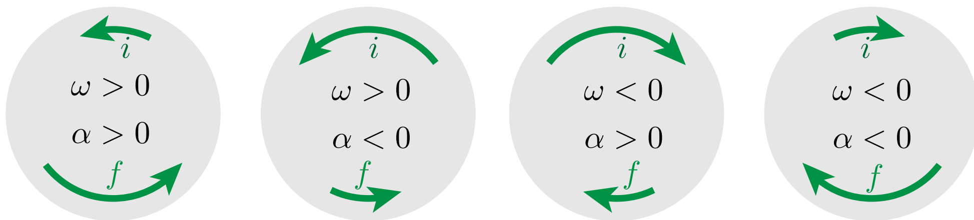
Sign Conventions

Choosing a positive and negative was easy in linear motion.

For angular velocities, we need a convention:

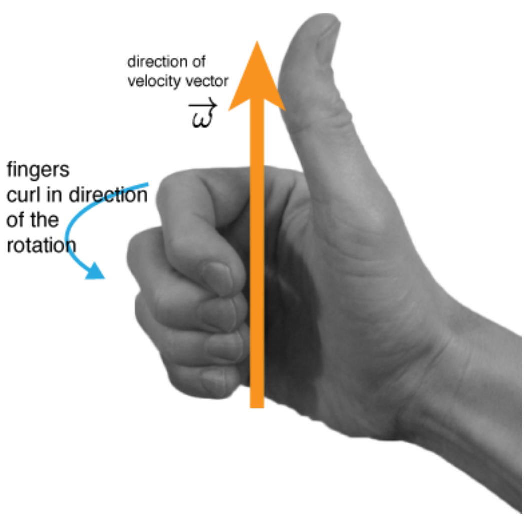


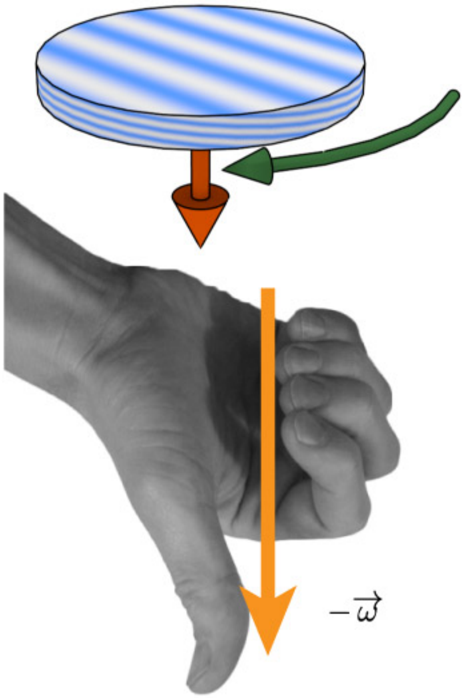
From this, we can also declare the sign of accleration:



2. Angular Velocity Vector

To describe the direction of an angular velocity, we'll need the **Right Hand Rule**.

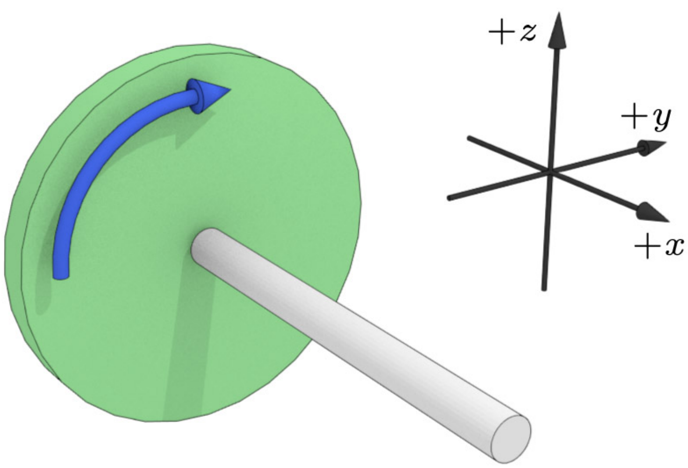




In the case of a negative $\boldsymbol{\omega}$, the same right hand rule can help you out.

Quick Question 2

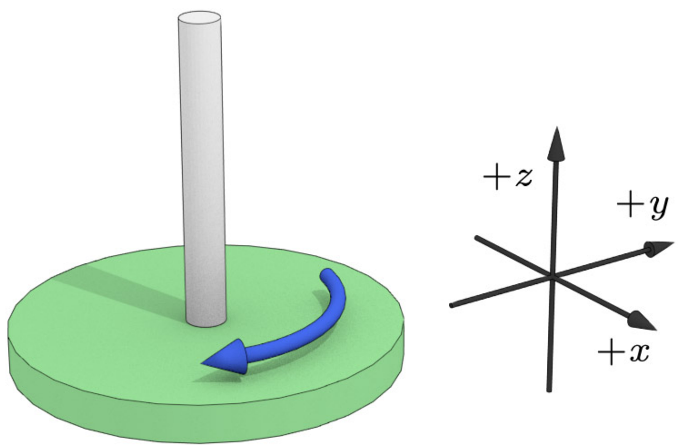
This wheel is rotating at constant angular velocity $\boldsymbol{\omega}$ in the direction of the blue arrow. What vector describes the direction of the angular velocity vector?



a) $+\mathbf{x}$
b) $-\mathbf{x}$
c) $+\mathbf{y}$
d) $-\mathbf{y}$
e) $+\mathbf{z}$
f) $-\mathbf{z}$

Quick Question 3

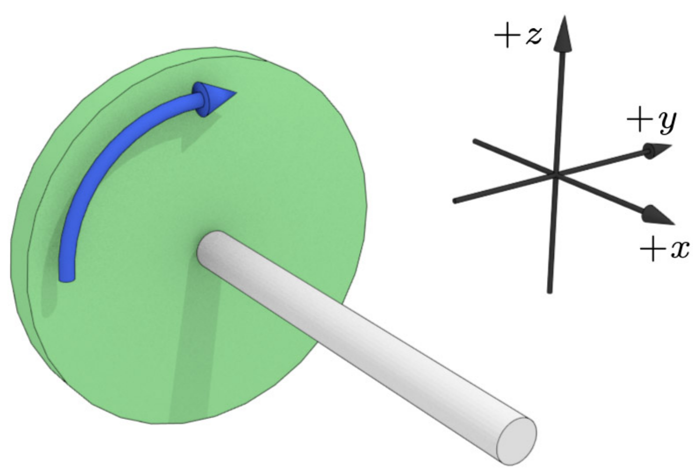
The wheel is rotating in the direction of the blue arrow, but is slowing down. Which direction is the angular acceleration vector pointing?



a) $+\mathbf{x}$
b) $-\mathbf{x}$
c) $+\mathbf{y}$
d) $-\mathbf{y}$
e) $+\mathbf{z}$
f) $-\mathbf{z}$

Quick Question 4

The wheel is rotating in the direction of the blue arrow, but is speeding up. Which direction is the angular acceleration vector pointing?



a) $+\mathbf{x}$
b) $-\mathbf{x}$
c) $+\mathbf{y}$
d) $-\mathbf{y}$
e) $+\mathbf{z}$
f) $-\mathbf{z}$

Quick Question 5

A wheel rotates at some angular velocity...

3. Rotational Kinematics

Fortunately, *they are all the same!*

Equations for linear motion	Equations for angular motion
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x = \dfrac{(v + v_0)t}{2}$	$\theta = \dfrac{(\omega + \omega_0)t}{2}$
$x = x_0 + v_0 t + \dfrac{at^2}{2}$	$\theta = \theta_0 + \omega_0 t + \dfrac{\alpha t^2}{2}$
$v^2 = v_0^2 + 2a_x x$	$\omega^2 = \omega_0^2 + 2\alpha\theta$

Must Use Radians!

> Example Problem #1:
>
> An old fashioned computer disk spins up to 5400 rpm in 2.0 s. Find the angular acceleration of the disc. After two seconds, how many revolutions has it made?

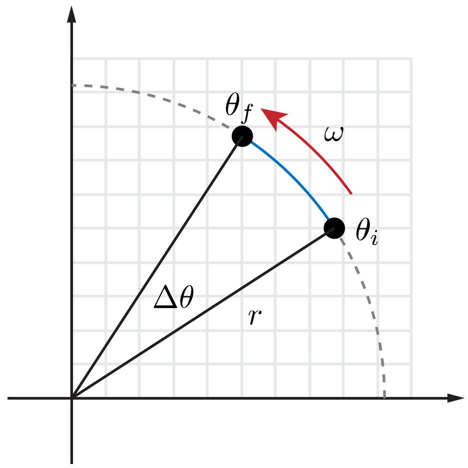
Quick Question 6

Which point on this record has the largest angular velocity ($\boldsymbol{\omega}$)?

a) A
b) B
c) C
d) They are all equal

Example Problem #2:

The propeller of an airplane is at rest when the pilot starts the engine; and its angular acceleration is a constant value. Two seconds later, the propeller is rotating at 10π rad/s. Through how many revolutions has the propeller rotated through during the first two seconds?



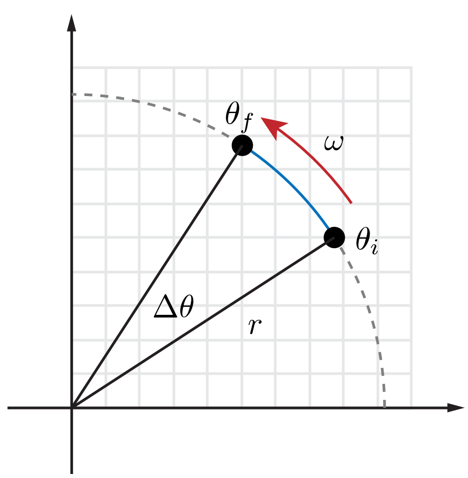
We can ask about the tangential velocity or speed of an object going around in circles.

Since $s = r\theta$ (in radians):

$$\frac{s}{t} = \frac{r\theta}{t} = r\omega$$

which will give us a tangential velocity: v_T.

$$v_T = r\omega$$



Similarly, we can ask about the tangential angular acceleration in the same way:

$$a_T = \frac{v_T - v_{T0}}{t} = \frac{r\omega - r\omega_0}{t} = r\left(\frac{\omega - \omega_0}{t}\right)$$

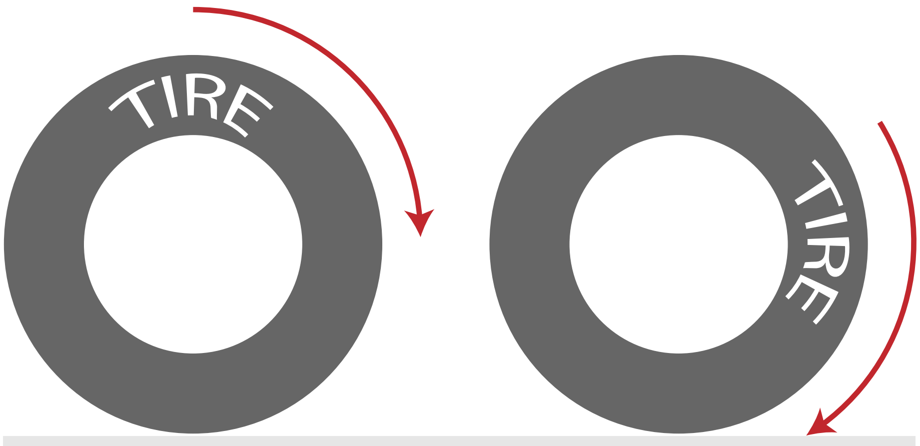
$$a_T = r\alpha$$

Quick Question 7

Which vectors are being displayed here?

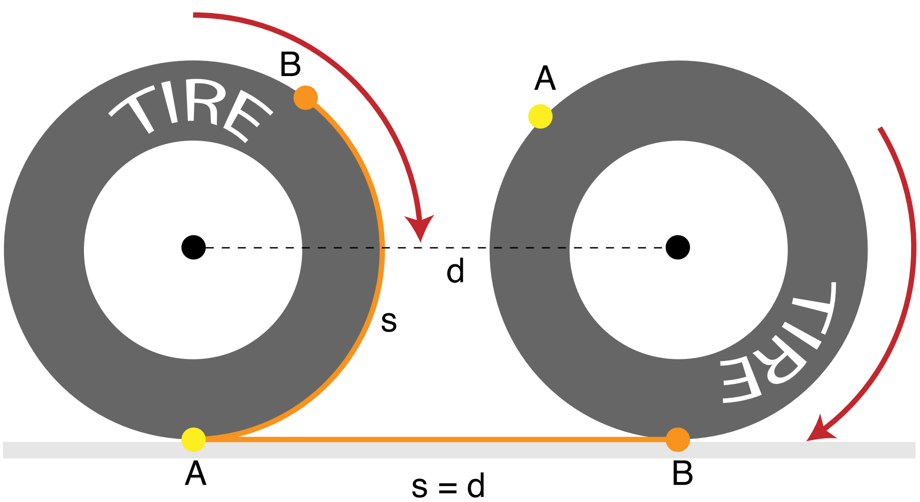
a) Angular velocity, ω
b) Tangential velocity, v_T
c) Centripetal acceleration, a_c
d) Tangential acceleration, a_T
e) Angular acceleration, α

Rolling Objects



If an object is rolling, then it will undergo both rotational motion *and* translational motion.

Rolling Object



While the tire is rolling, the arc length s will be traced out along the ground.

If we consider the distance traveled by the tire, d, we can see that it must be the same as s. Thus:

$$v_{cm} = r\omega$$

likewise for the acceleration:

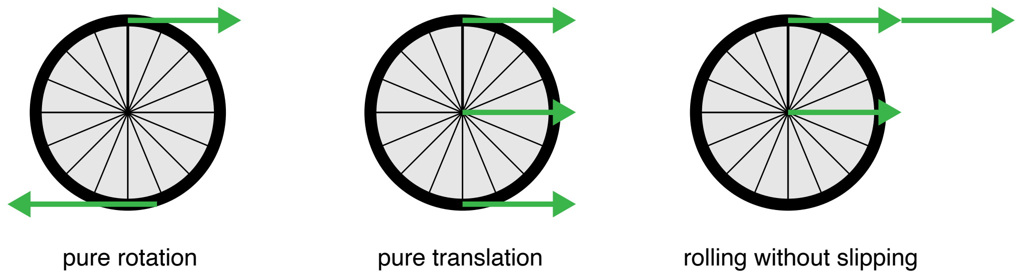
$$a_{cm} = r\alpha$$

This condition is often called **rolling without slipping**.

A bicycle is moving forward with a speed v_0. At any moment in time, what is speed of the lowermost point of one of the wheels?

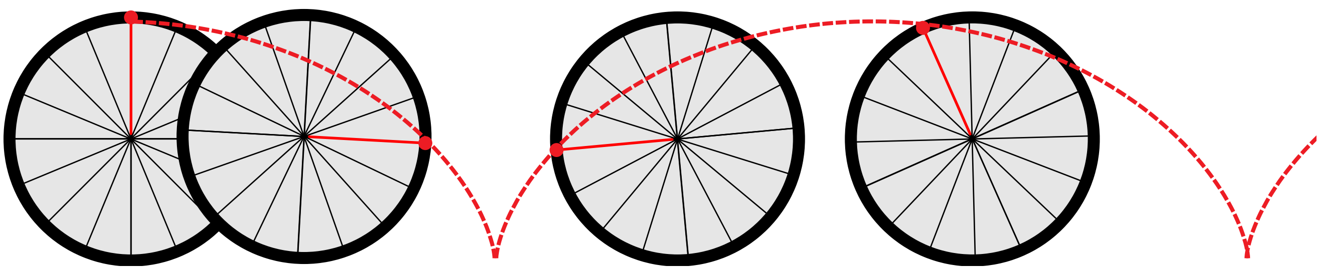
Rolling without slipping

Vector addition still works



Rolling

A point at the rim of a rolling wheel, when not slipping at all, will trace out the following curve:



It's known as a cycloid.

Angular momentum

Linear momentum was given by the mass times the velocity:

$$p = mv$$

Angular momentum will be given by the analogous terms:

$$L = I\omega$$

You'll note the letter I in the above equation for L, the angular momentum. This I is known as the moment of inertia and serves as the analogue for mass in rotational systems. We'll investigate it more in the next chapter.

Which of the following coordinate systems is (are) a valid right hand coordinate system?