

Fluids

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Quick Question 1

(Que)estimate the mass of the air in this room.

- a) 1 gram (two raisins)
- b) 100 grams (an orange)
- c) 1 kilogram (A Chihuahua)
- d) 10 kilograms (A basket full of Chihuahuas)
- e) 100 kilograms (Two ballet dancers)
- f) 1000 kilograms (A small car)
- g) 10000 kilograms (Very large truck)

The point here is to just guess how much the air weighs. No calculations, no estimates, just give it a guess.

1. What is a fluid?

A fluid is any material that can flow.

Flow means to deform under an applied stress.

Many things can be classified as fluids: air, water, oil, syrup, etc.

All the objects we've encountered so far were non-deformable. They could be moved or rotated, but their shape would never really change. If the shape of an object can be continuously deformed due to an applied stress, then we have a different situation. This is the domain of fluid mechanics. "Fluid", in common language, refers to liquids. However, in the eyes of physics, a fluid can be a liquid, gas, collection of particles.

2. Mass density

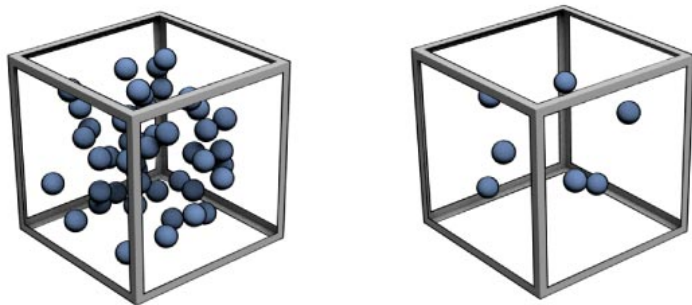
To describe these materials, we'll often use the density, ρ , which is defined as the mass per unit volume:

$$\rho = \frac{m}{V}$$

Clearly, water will be denser than air, since 1 cubic meter of water has a much larger mass than 1 cubic meter of air.

Air: 1.2041 kg/m^3 and water: 1000 kg/m^3

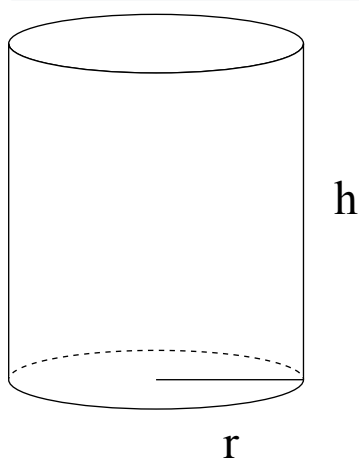
different densities



Estimate the mass of the air in this room

We just need to estimate the volume of the lecture hall. It's kinda like a cube. Maybe $6\text{w} \times 6\text{h} \times 20\text{l}$? So a volume of 720 cubic meters. If air has a density of 1.29 kg/m^3 , then the mass of the air is 928 kg, approximately 1000 kg, or 2,200 lbs.

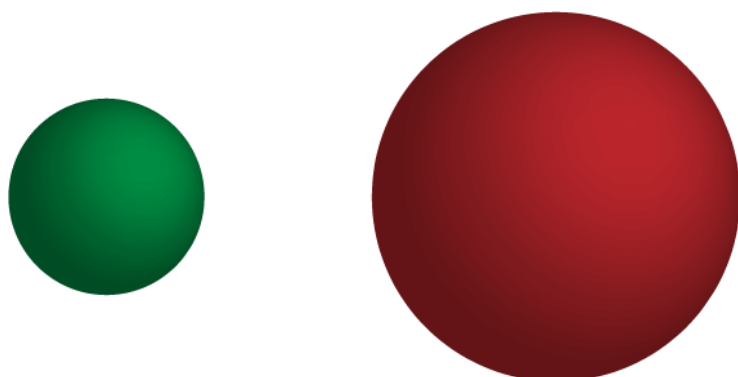
Quick Question 2



What is the volume of this cylinder?

- a) $4\pi r h$
- b) $\pi r h$
- c) $3\pi r^2 h$
- d) $\pi r^2 h$
- e) $\pi r^2 h^2$

Quick Question 3



Here are two spheres. The red one has twice the radius and twice the mass of the green one. How do their densities compare?

- a) The Red one is four times denser
- b) The Red one is twice as dense
- c) They both have the same density
- d) The Green one is twice as dense
- e) The Green one is four times denser

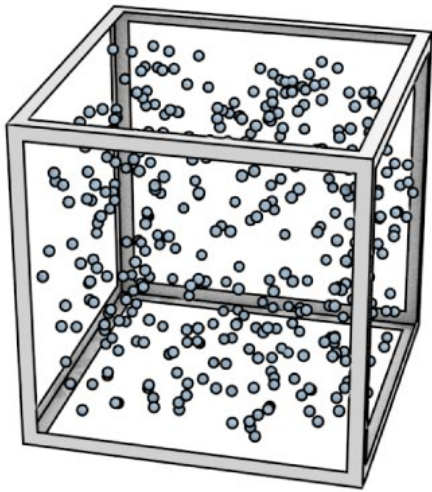
Example Problem #1:

A cylinder is used to hold some air that has a density of 1.5 kg per cubic meter. If the radius of the cylinder is increased by 50%, what will the density of the air be?

In a fluid, the constituent particles are continuously in motion.

We saw how the motion of a ball hitting a wall could be analyzed in terms of impulse and momentum. In a fluid, there are little balls (e.g. air molecules or water molecules) hitting the walls all the time. Lot's of them. They don't go very far however, because they collide with other balls very often.

3. Pressure



Imagine a box with 6 surfaces, each with area A . Inside the box are many air molecules all bouncing all over the place.

We can define the pressure, P , of the air in the box by considering the amount of force exerted by all these collisions on a surface of the box:

$$P = \frac{F}{A}$$

If there are more collisions in a given time, then there will be a greater force, and thus a larger pressure.

Pressure

So, pressure is the force per unit area.

The SI unit is called the Pascal:

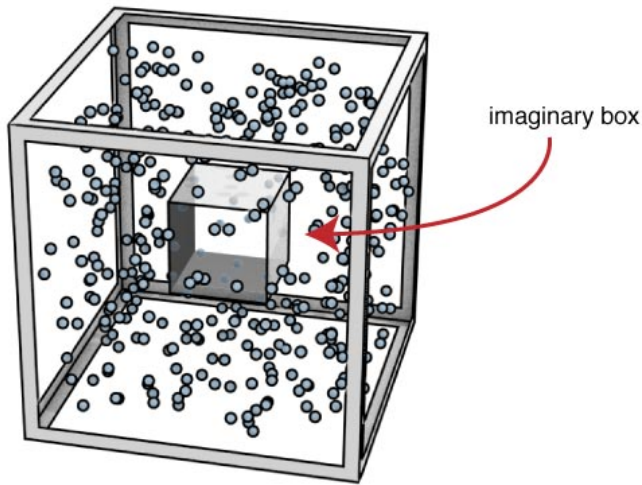
1 Newton per 1 meter² = 1 Pascal

This turns out to be not very much pressure.

We'll see that more common amounts of pressure would be several tens of thousands of Pascals: 1×10^5 Pa for example.

PSI (pounds per square inch) is also a *force per area*. That unit is more common in everyday life.

1 Atmosphere is another unit of pressure (atm) and is equal to 1.015×10^5 Pa. The air molecules above are exerting that much pressure on us right now.



Now, we can imagine a little cube inside this space.

Each of the faces of this little cube will experience a force directed perpendicular to it.

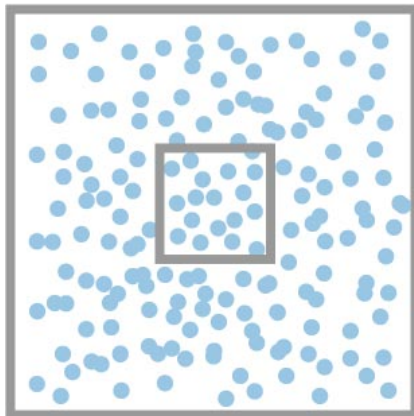
The pressure on the little internal box will be the same as it is on the external walls.

$$P_{\text{little box}} = P_{\text{big box}}$$

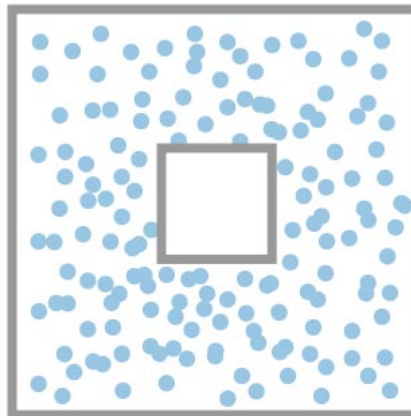
because the surface areas of the box are smaller, but the number of collisions occurring on the little box is also smaller.

Now we can imagine two scenarios:

box is filled with equal pressure air

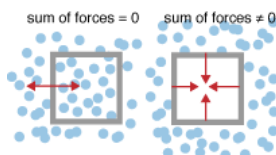


box is empty (vacuum)



What would the forces look like on these boxes?

In the case where there are no (or less) particles of fluid inside the small box, there will be no outward directed forces on the box. This explains why when you let the air out of a balloon, it collapses.



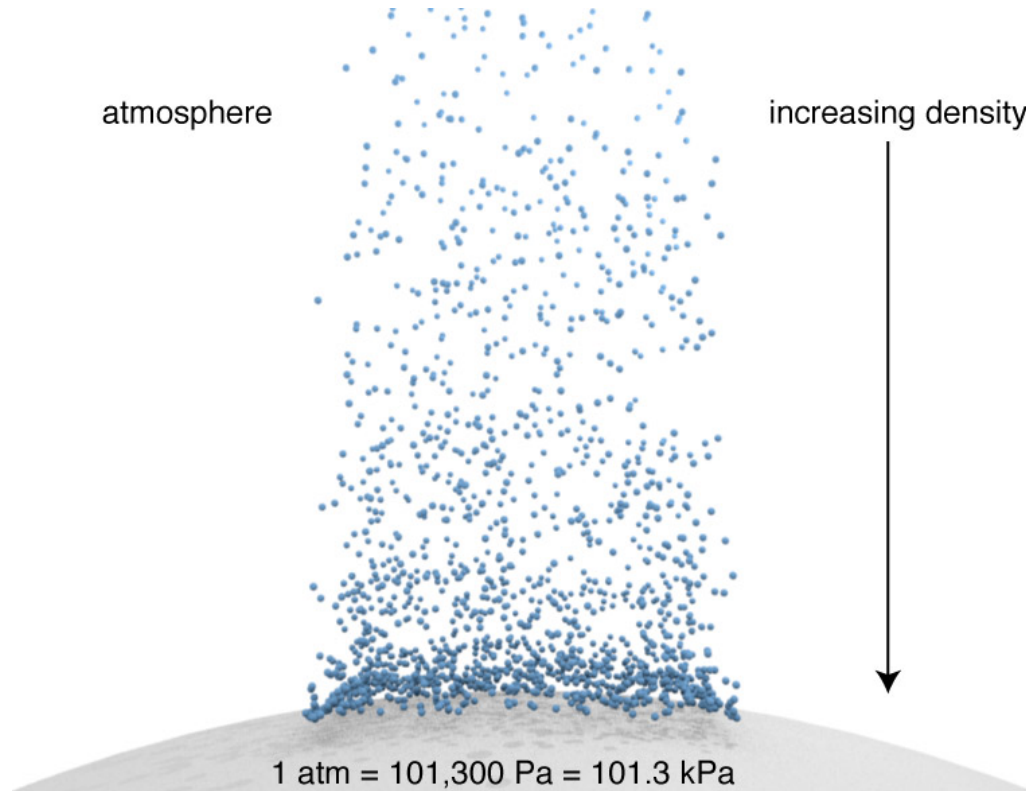
3.1 Compressible vs. Incompressible

In a gas, like air, the atoms have a lot of space between them. Thus, by applying a force to the container the air is in, we can *compress* the fluid by a significant amount.

In the case of water, the atoms are packed together about as close as possible. This makes it much harder to compress the fluid. Such a fluid is called *incompressible*.

This is mentioned as a way of introducing atmospheric pressure.

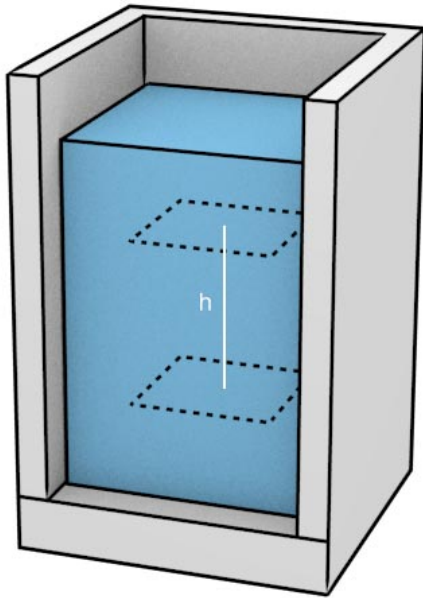
3.2 Atmospheric Pressure



Due to gravity, the pressure close to the earth will be larger.

Whenever we talk about the atmospheric pressure, we are referring to the forces felt by all the little collisions of air molecules. Because of earth's gravity, there will be more molecules near the surface of the earth, than high above it. This make the air pressure decrease as we go up in elevation. So, for most problems, when we say 'atmospheric pressure', we mean the pressure near the surface of the earth: $1.013 \times 10^5 \text{ Pa}$.

Pressure and depth



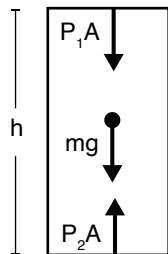
Now let's consider a column of water surrounded on all sides (4th side not shown)

What would the pressure difference be between the two dotted squares?

A free body diagram shows that:

$$\sum F_y = P_2 A - P_1 A - mg = 0$$

since the mass is not accelerating. We can use the fact that $\rho = m/(Ah)$ to make this expression easier to use.



Let's use our old friend, the free body diagram, to assess the situation. Imagine a section of the water bounded on the top by the two dashed squares. A f.b.d. would have two forces acting on the top and bottom sides. There would also be forces acting from the left and right, but we can neglect those right now. The third force that is acting the vertical direction is the force of gravity, mg . It acts at the center of mass of this imaginary shape. The magnitude of the forces acting in the vertical direction is given by: $F = P_h A$, where P_h is the pressure at the given height.

3.3 Pressure and depth

Thus, the pressure difference between two points in a vertical column of water (or other fluid) can be figured out by the following:

$$P_2 = P_1 + \rho gh$$

If we know the pressure at point P_1 , then we can use the density and the depth of the fluid below to find the pressure there.

This assumes that the ρ doesn't change much with the depth. For an incompressible fluid like water, this is a good assumption.

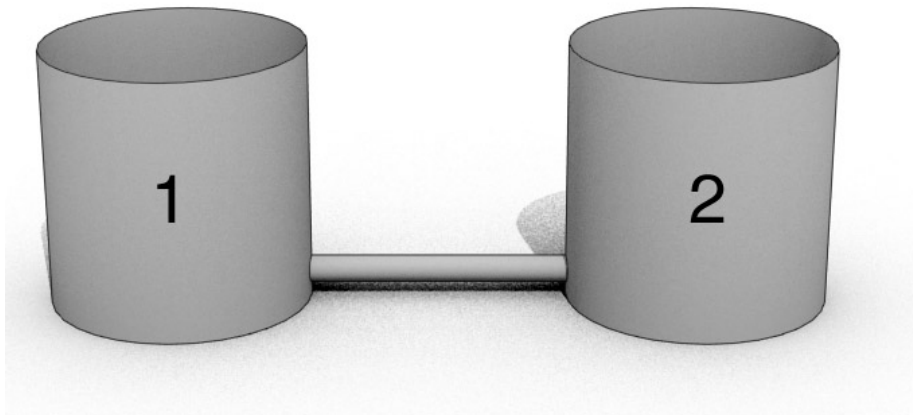
If we are dealing with a fluid like air, then this equation would only be valid for small changes in h , where the density of the air did not change much.

Example Problem
#2:

A submarine dives to 300m. What is the pressure at this depth? (In Pa, and atm) (Density of sea water is 1030 kg/m^3)

Quick Question 4

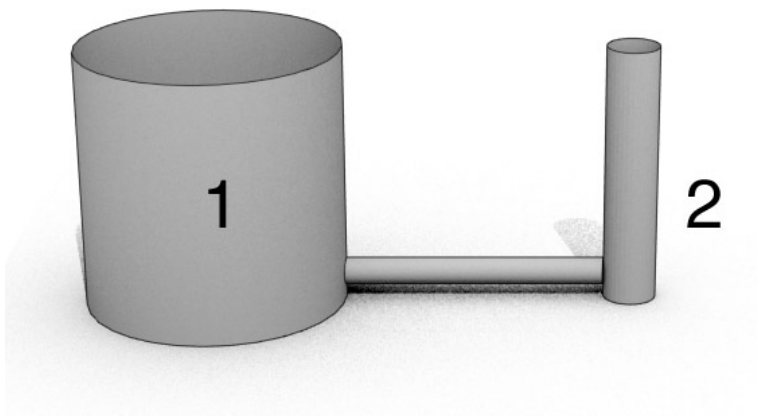
Here are two vessels connected by a tube at the bottom. If we pour water in bucket 1, how high will the water go in bucket in 2?



- a) bucket 2 will be at a higher level
- b) bucket 2 will be at the same level
- c) bucket 2 will be at a lower level

Quick Question 5

Here are two vessels connected by a tube at the bottom. If we pour water in bucket 1, how high will the water go in bucket in 2?



- a) bucket 2 will be at a higher level
- b) bucket 2 will be at the same level
- c) bucket 2 will be at a lower level

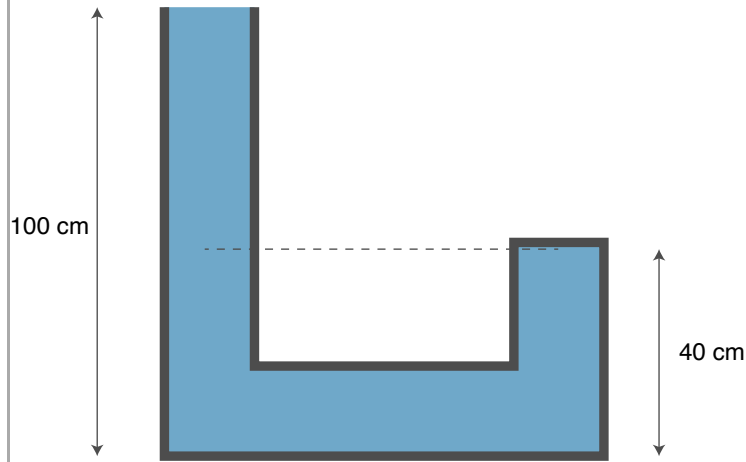
Major Consequence

In light of this relation, we can see that there is no mention of a horizontal component.

$$P_2 = P_1 + \rho gh$$

This implies that the pressure along any horizontal line in a connected fluid is the same.

Example Problem
#3:



What is the pressure at the closed end of this tube, which is filled with water ($\rho = 1000 \text{ kg/m}^3$)?

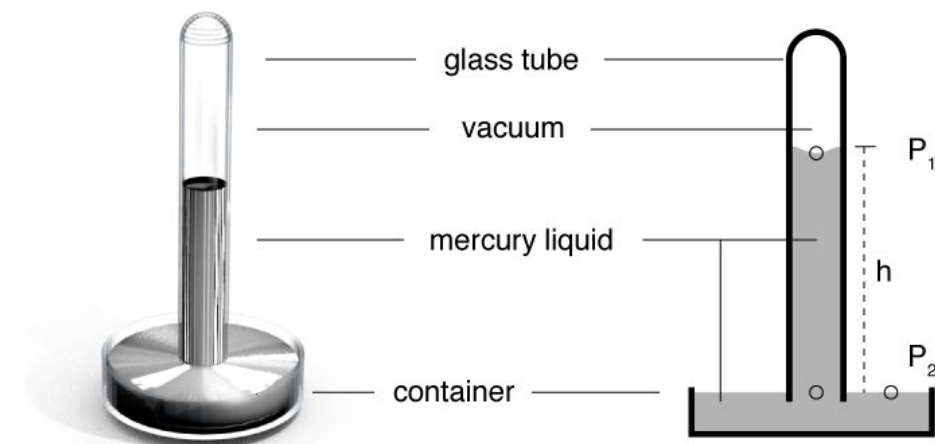
Quick Question 6

About how far below the surface of a lake would you have to go to have a pressure equal to twice the pressure at the top? (the density of water is 1000 kg/m^3 and atmospheric pressure is $1 \times 10^5 \text{ Pa}$)

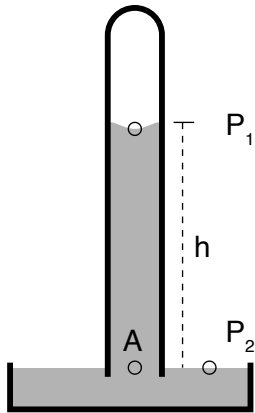
- a) 1 meter
- b) 2 meters
- c) 10 meters
- d) 20 meters
- e) 100 meters

3.4 Pressure Gauges

The mercury barometer



A tube, with a vacuum inside is placed upside down in a vat of mercury. The air pressure on the surrounding fluid causes the height of the Hg in the tube to rise.



Since the pressure is zero above point P_1 , we can say $P_1 = 0$. And, since the pressure is the same along any horizontal line, we can also say the pressure at points A and P_2 are the same. Thus, we can use the depth/pressure equation to calculate the pressure at P_2 based on h , the height of the mercury in the tube.

$$P_1 = 0$$

$$P_2 = 0 + \rho gh$$

And, since P_2 is just the atmospheric pressure: $P_2 = P_{\text{atm}}$, we can write:

$$P_{\text{atm}} = \rho gh$$

ρ and g are known and we can measure h .

Units of pressure?



60°F

FEELS LIKE 60°

Fair

*Most recent low
temperature much
above average.*

Past 24-hr Precip:
0.09 in (est.)

Wind:

→ W at 11 mph
gusting to 21 mph



65°

HIGH AT 1:05 PM

Sunny



40°

LOW

Clear

Chance of Rain

💧 0%

Wind:

→ W at 19 mph

Humidity:
27%

UV Index:
0 - Low

Next 6 Hours:
Clear with temperatures
falling into the upper
40s. Winds W at 15 to
20 mph. [Hourly Details](#)

Pressure:
29.73 in 

Dew Point: 
26°

Visibility:
10.0 mi

Humidity:
43%

Tonight:
Mostly clear. Low near
40F. Winds W at 15 to
25 mph.

Sun & Moon

Sunrise:
6:46 am

Moonset:
7:33 am

Moonphase:
 Full

Sunset:
4:35 pm

Moonrise:
5:35 pm

The units that are reported are often a length, which is certainly not a force per unit area. (N/m^2)

But, that's ok because the density of mercury doesn't change much, and neither does g .

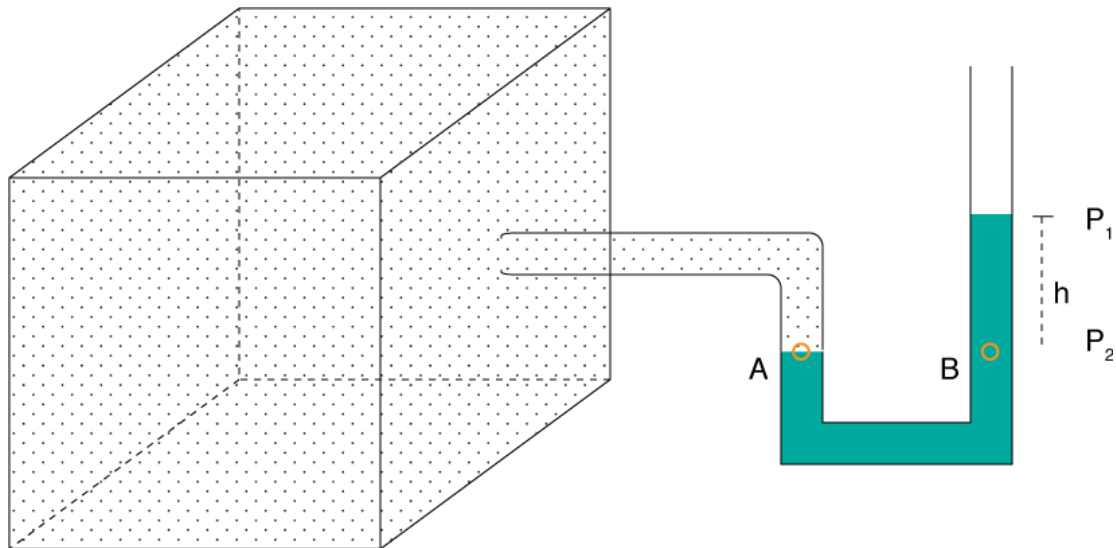
$$\rho_{\text{Hg}} = 13.6 \times 10^3 \text{ kg/m}^3$$

$$g = 9.8 \text{ m/s}^2$$

Thus, we can just report the units value of h , and everyone around the US will probably be ok with that.

Centimeters of Hg are also sometimes used.

Manometer



A similar tool is the manometer.

The pressure at A can be determined by the height h . If the height h is zero, then the fluid in the box has the same pressure as the atmosphere. If not, then it will be different.

$$P_2 - P_{\text{atm}} = \rho gh$$

This tells us how different the pressure is inside the box, compared to the atmospheric pressure.

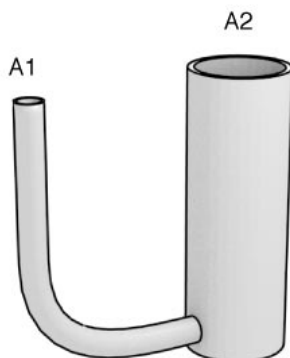
Quick Question 7

When we use a drinking straw, which force causes the water to go up the straw?

- a) that due to a low pressure region caused by sucking
- b) that due to the pressure within the liquid
- c) that due to atmospheric pressure
- d) that due to the person sucking on the straw
- e) that due to friction forces within the straw

4. Pascal's Principle (or law)

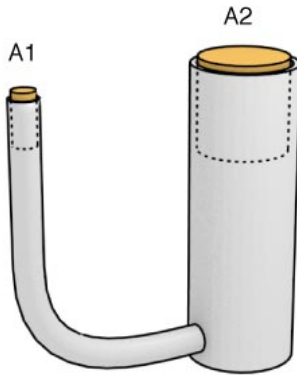
"Any change in the pressure applied to a completely enclosed fluid is transmitted undiminished to all parts of the fluid and the enclosing walls."



If the heights of two columns are the same, then P_1 has to equal P_2 .

And since $P = F/A$, we can find the relationship:

$$F_2 = F_1 \left(\frac{A_2}{A_1} \right)$$



So, if we put pistons in both tubes, such that pressing on one will rise the other, we can see that based on the ratios of the areas involved, a small force on the small piston will result in a large force on the large piston.

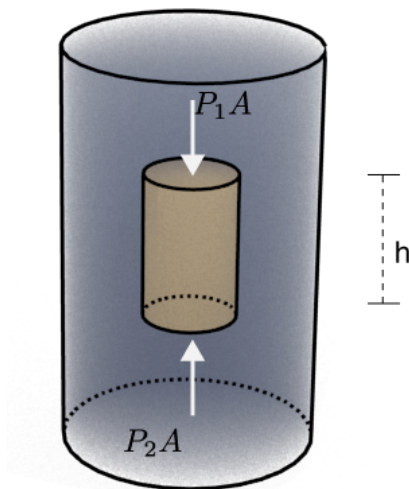
This is the mechanism behind the hydraulic jack.

Quick Question 8

What will the scale do as I dip my finger into the water?

- a) The side with the water will go down
- b) They will stay equal
- c) The side with the water will go up

5. buoyant Force



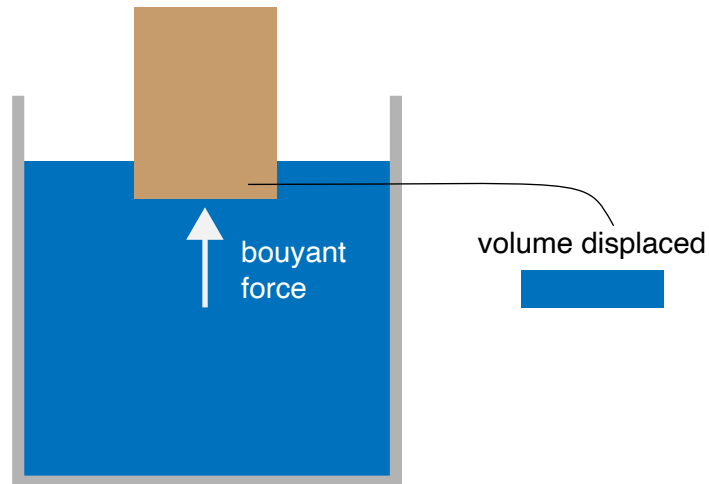
Consider a cylinder submerged in a vessel of fluid.

Since the pressure is higher at lower depths, the force on the bottom surface will be larger than the upper surface.

This difference in forces is known as the **buoyant force**, F_B .

$$F_B = P_2A - P_1A = (P_2 - P_1)A = \rho ghA$$

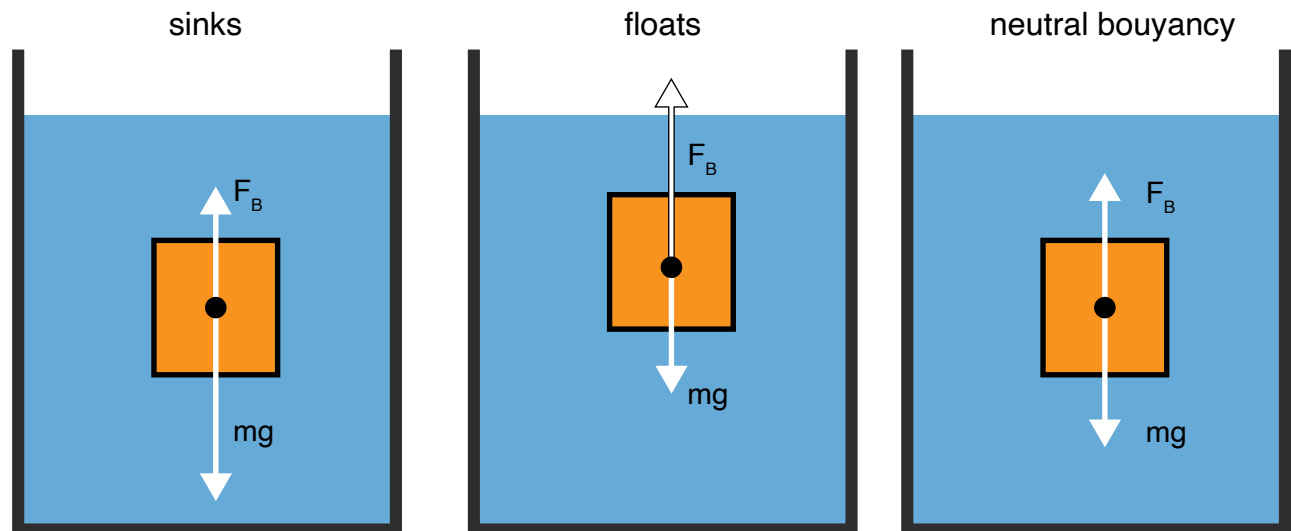
5.5 Archimedes' Principle



If an object is submerged (or partially) submerged in a fluid, the magnitude of the buoyant force equals the weight of the fluid displaced.

$$F_B = W_{\text{fluid}}$$

Float or sink?



Comparing the weight of the object with the Buoyant force will tell us if the object floats or sinks.

Quick Question 9

What will the scale read when I submerge the mass into the water?

- a) Less than 18 N
- b) 18 N
- c) More than 18 N

Example Problem #4:

Does it float

Determine if this golf ball will float. (Mass = 45.33 grams, diameter = 42.67 mm)

Example Problem #5:

Floating Balloons

Helium gas has a density of $.166 \text{ kg/m}^3$. If the mass of an empty balloon is 2.0g, how large (i.e. its diameter) should we blow it up to achieve neutral buoyancy in air?

Quick Question 10

Holding a piece of wood underwater requires a certain amount of force. How does this force depend on the distance below the surface of the water?

- a) Closer to the surface will require less force.
- b) Deeper under the water will require less force.
- c) The force will be the same everywhere

Quick Question 11

What happens when a certain volume of water freezes?

- a) It gets bigger
- b) It gets smaller
- c) It stays the same volume

Quick Question 12

What happens as water freezes?

- a) Its density increases
- b) Its density decreases
- c) Its density stays the same.

Quick Question 13

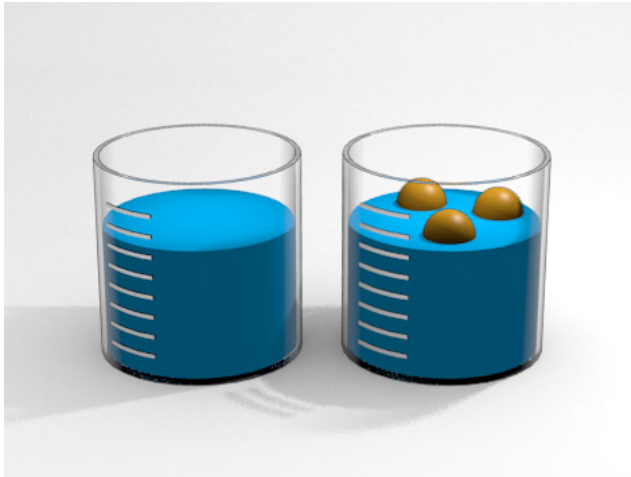
An ice cube is floating in a glass of water that is filled entirely to the brim. When the ice cube melts, the water level will:

- a) Fall
- b) Stay the same
- c) Rise, causing the water to spill

Example Problem #6:

A block of wood floats in a fresh water lake. If the block of wood is 10 cm on each side, how much is sticking out above the water? (i.e. how far is it from the top of the water to the top of the wood block) ($\rho_{\text{wood}} = 700 \text{ kg/m}^3$)

Quick Question 14



Which bucket, when placed on a scale, would weigh more? The water level in both is identical.

- a) The bucket with no oranges
- b) They would weigh the same
- c) The bucket with three oranges
- d) Cannot be determined

Example Problem #7:

A "gold" crown weighs 8.30 Newtons. We hang the crown from a string and submerge it in water. The tension on the string is measured to be 7.81 N. Is the crown actually gold? (The density of Gold is known to be $19,300 \text{ kg/m}^3$)

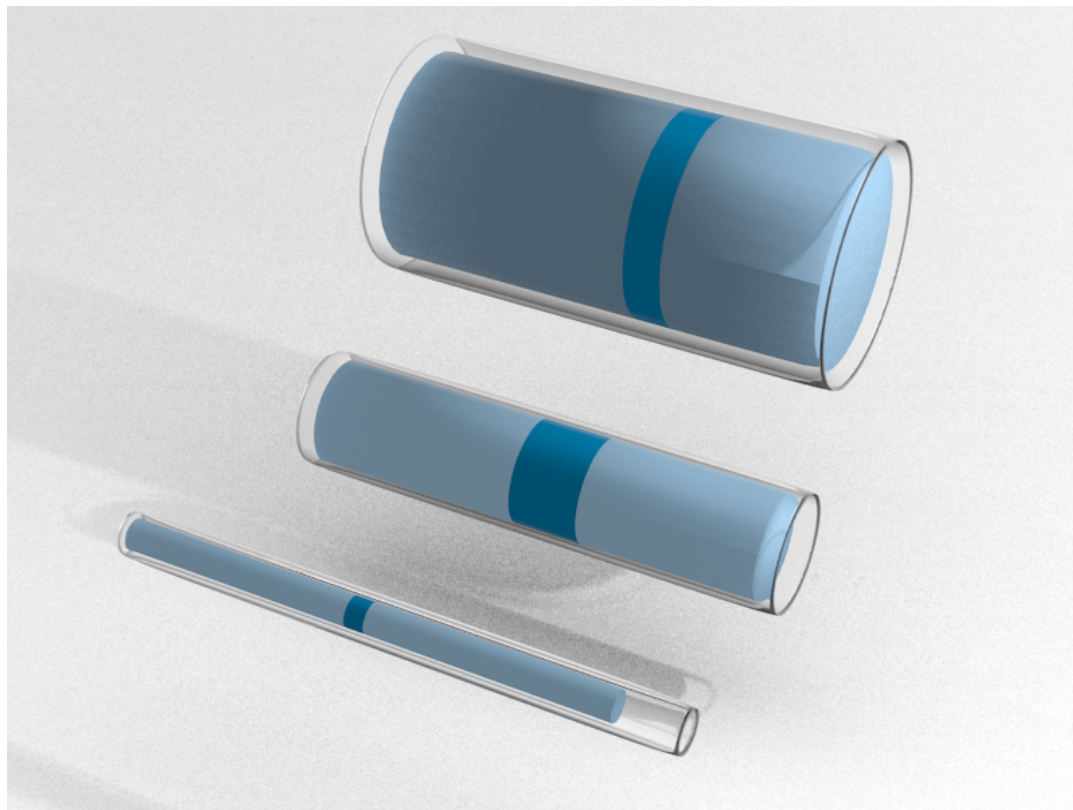
5.6 Moving Fluids

6. Moving Fluids

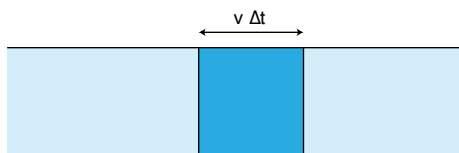
Assumptions (to make things doable)

1. Steady Flow (laminar): The velocity of the fluid at any point does not change in time.
2. Incompressible: The density of the fluid is constant.
3. Non-viscous Flow: The fluid doesn't provide any resistive forces.

6.7 Mass Flow Rate



We can figure out **how much stuff is flowing** through a given pipe in a time t .



The distance any fluid travels in a tube is given by: $\Delta x = v\Delta t$.

The volume of the fluid that passes through that length can be figured out by:

$$V = A\Delta x = Av\Delta t$$

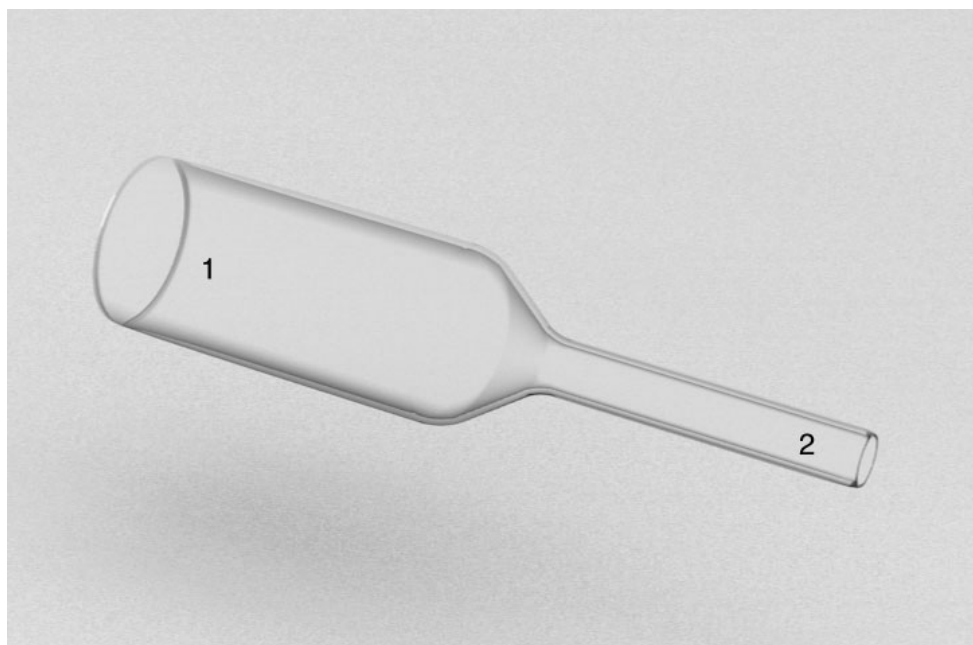
where A is the cross sectional area of the tube.

Thus, since $\rho = m/V$, the mass that flows is given by:

$$\Delta m = A\rho v\Delta t$$

Mass flow rate

The amount of stuff passing through point 1 in a given time better be the same as the amount that passes through point 2.



6.8 The Continuity Equation

$$\frac{\Delta m}{\Delta t} = \rho_2 A_2 v_2$$

$$\frac{\Delta m}{\Delta t} = \rho_1 A_1 v_1$$

The mass flow rate has the same value at every position along a tube, provided the tube only has a single exit or entrance.

$$\frac{\Delta m}{\Delta t} = \rho_2 A_2 v_2$$

$$\frac{\Delta m}{\Delta t} = \rho_1 A_1 v_1$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

If the density is same (i.e. an incompressible fluid), then:

$$A_1 v_1 = A_2 v_2$$

We can also talk about the volume flow rate:

$$Q = Av$$

Example Problem
#8:

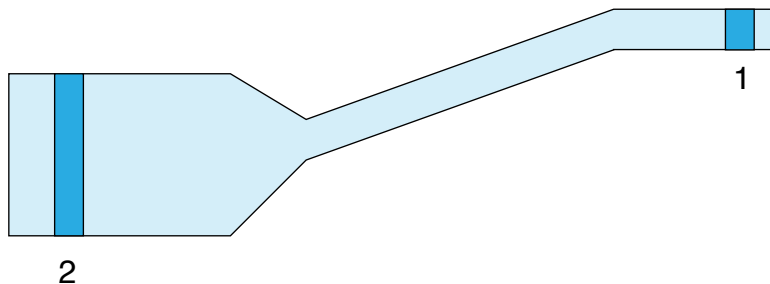


Water pours smoothly from a faucet. Gravity causes the water to speed up as it falls. Use the continuity equation to find the volume flow rate. The two measured cross-sectional areas are: 1.2 cm^2 and 0.35 cm^2 and they are separated by a distance of 45 mm.

Example Problem
#9:

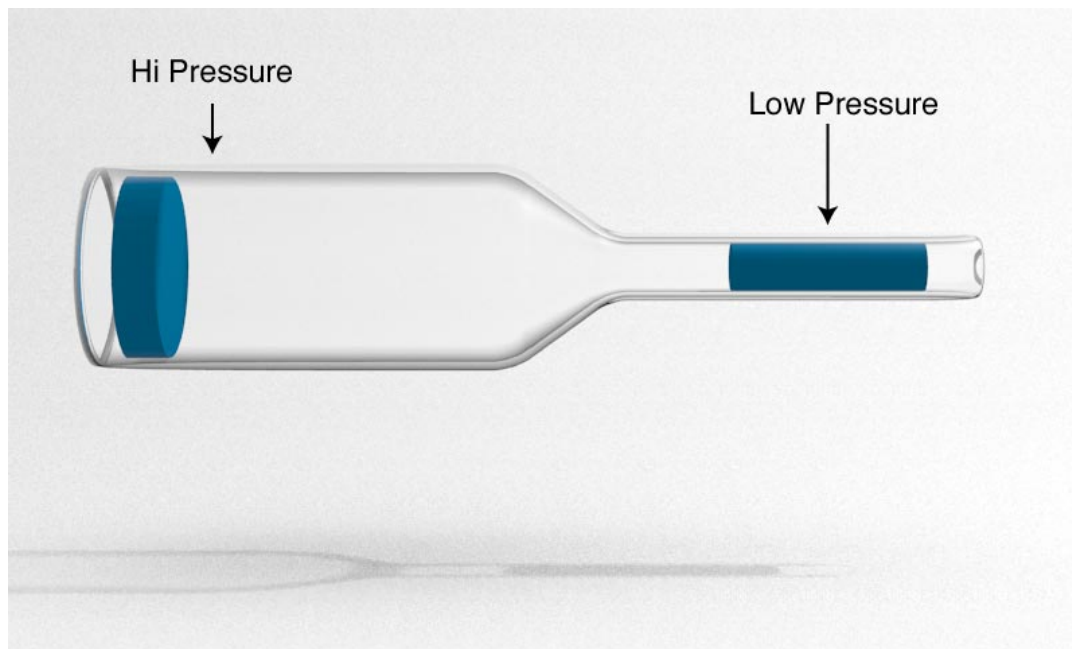
A garden hose has an inner diameter of 1.6 cm. It takes 20 s to fill a 10 L bucket with it. What is the speed of the water as it leaves the end of the hose?

What about if there are changes in diameter, and elevation?

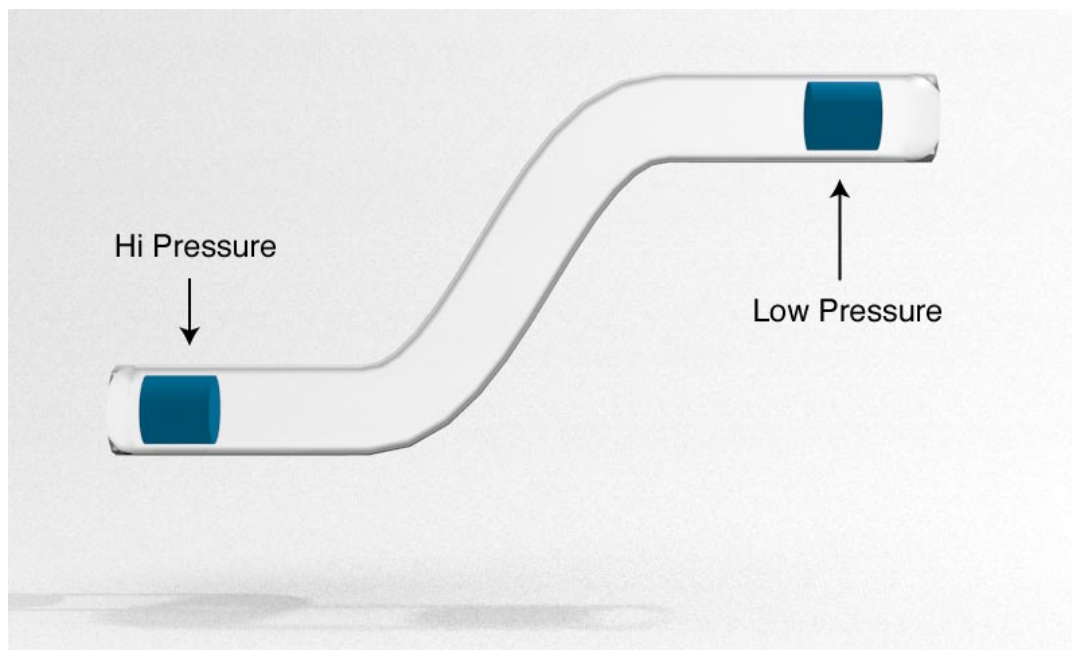


Bernoulli's Principle

For a moving fluid, what is the pressure difference between these two regions?



For a moving fluid, what is the pressure difference between these two regions?



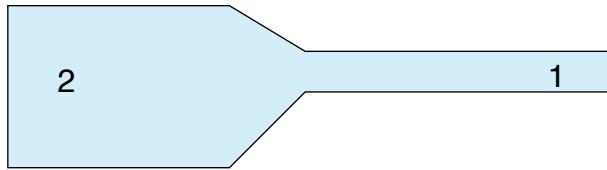
Bernoulli's Equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

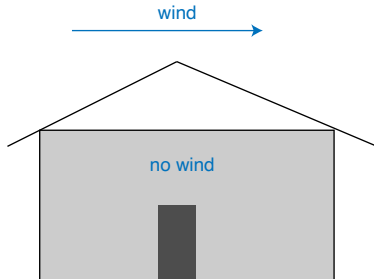
This equation shows the relationship between Pressure, velocity, and elevation of a moving fluid.

If we don't worry about elevation changes, $y_1 = y_2$.

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$



Applications of Bernoulli's Equation

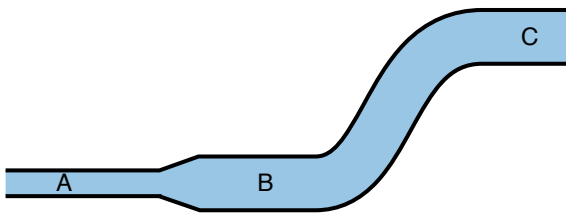


Since the pressure above the roof can be lowered by the velocity of the wind, there will be an upward force on the roof from the high pressure air inside the house.

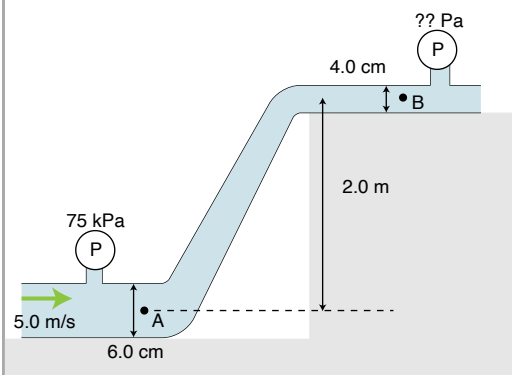
Quick Question 15

Fluid is flowing from left to right through the pipe shown in the drawing. Points A and B are at the same height, but the cross-sectional areas of the pipe are different at the two locations. Points B and C are at two different heights, but the cross-sectional areas of the pipe are the same at these two locations. Which of the following is true?

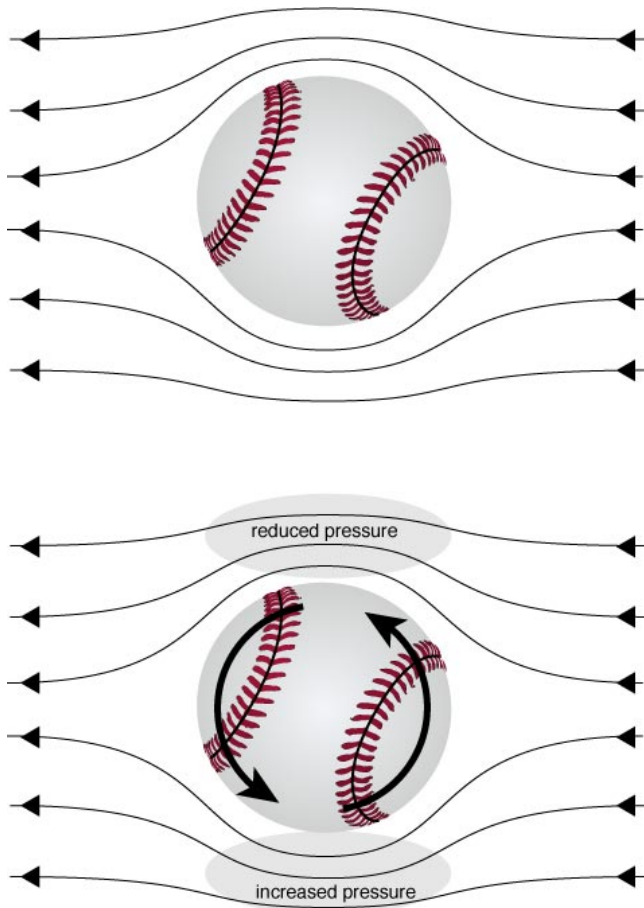
- a) $P_A > P_B > P_C$
- b) $P_B > P_A = P_C$
- c) $P_C > P_B > P_A$
- d) $P_B > P_A$ and $P_B > P_C$
- e) $P_C > P_A$ and $P_C > P_B$



Example Problem #10:



Find the pressure at point B for this pipe full of flowing water.

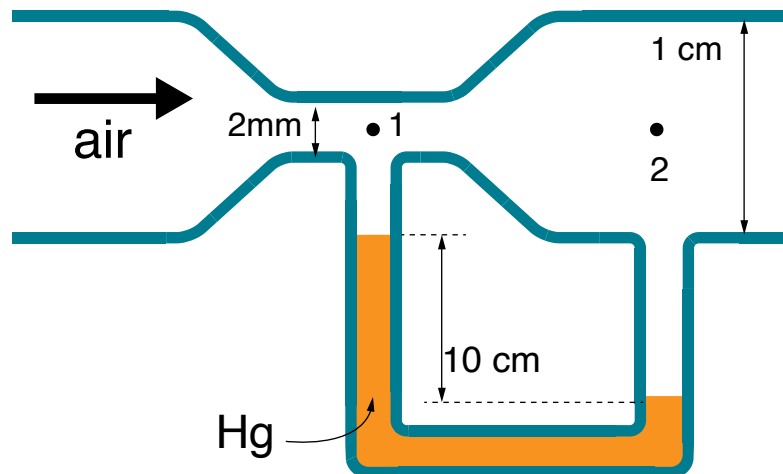


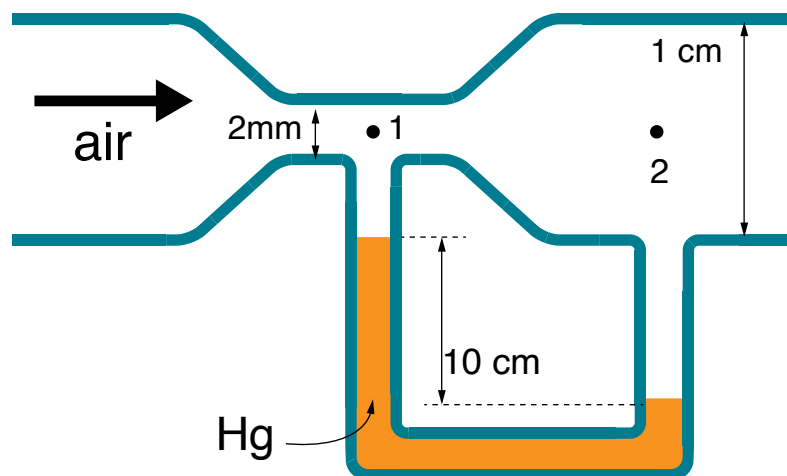
If a baseball, or any ball is thrown with out spin, then it will have the same pressure on all sides.

If however, we give the ball a large spin as it travels, the air in the immediate region of the ball will spin a little bit also. Thus, the air speed around the ball will be different on one side compared to the other.

This pressure difference will lead to forces on the ball which will cause it to venture off the straight line it would normally take.

Air flow through the tube below. What is the velocity of the air at point 1 and 2? (The liquid below is Hg, with a density of 13.6 kg/m^3).





A small hole opens at the bottom of a water tank. If the hole is a distance h below the surface of the water, what will the velocity of the water spurting out of the tank be?