Kinematics in One Dimension

1. Introduction

Motion: change in position or orientation with respect to time.

Vectors have given us some basic ideas about how to describe the position of objects in the universe/ Now, we'll continue by extending those ideas to account for changes in that position. Of course the world would be awfully boring if the position of everything was constant.

1.1 Different Types of Motion We'll look at:

Linear motion involves the change in position of an object in one direction only. An example would be a train on a straight section of the track. The change in position is only in the horizontal direction.

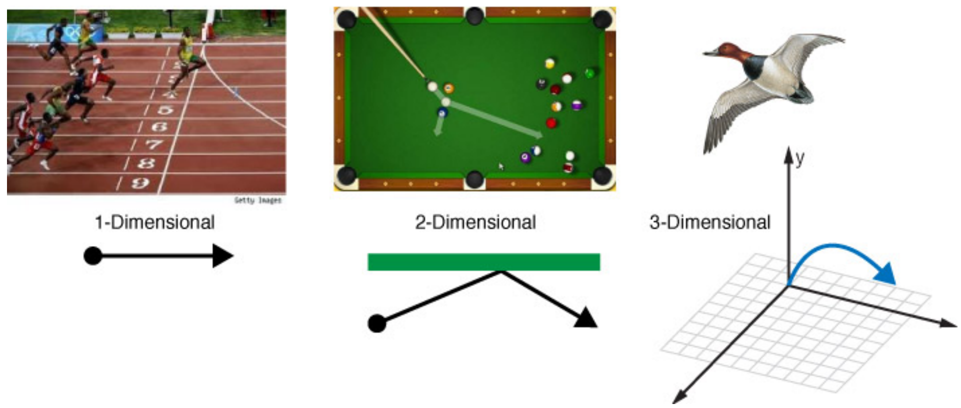
Projectile motion occurs when objects are launched in the gravitational field near the earths surface. They experience motion in both the horizontal and the vertical directions.

Circular motion occurs in a few specific cases when an object travels in a perfect circle. Some special math can be used in these cases.

Rotational motion implies that the body in question is rotating around an axis. The axis doesn't necessary need to pass through the object.

... or a combination of them.

1.2 Dimensionality in physics



There is another meaning to the word *dimension* in physics. This one is probably more familiar than the previous sense in which we used the word (to describe the unit of a given situation). It is related however. Our familiar world has 3 spatial dimensions. We often call them x, y and z. Or left/right, forwards/backwards, and up/down. If an object moves in only one of the directions, like say a train on straight track, then we would call this motion 1 dimensional translation. It is the easiest type of motion to analyze. Of course, few things only go in straight lines - even the trains have to turn to go around things. Then, the motion becomes 2 dimensional, since we require information about both the x and y motions.

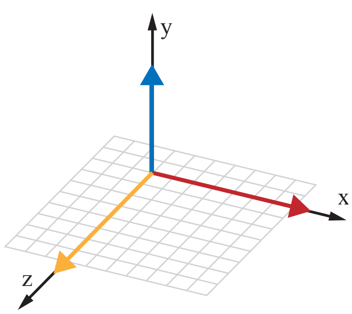
1.3 One dimensional kinematics

For the case of 1-dimensional motion, we'll only consider a change of position in one direction.

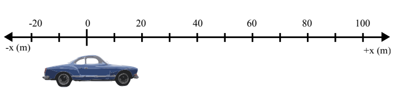
It could be any of the three coordinate axes.

Just a description of the motion, without attempting to analyze the cause. To describe motion we need:

1. Coordinate System (origin, orientation, scale)
2. the object which is moving



A coordinate system



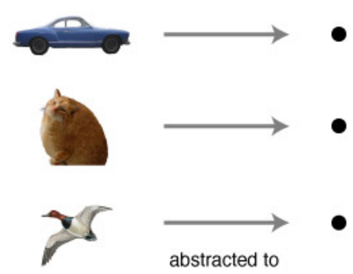
A 1-d coordinate system

1d kinematics will be our starting point. It is the most straightforward and easiest mathematically to deal with since only one position variable will be changing with respect to time.

1.4 Particle model

We'll need to use an abstraction:

All real world objects take up space. We'll assume that they *don't*. In other words, things like cars, cats, and ducks are just point-like particles.



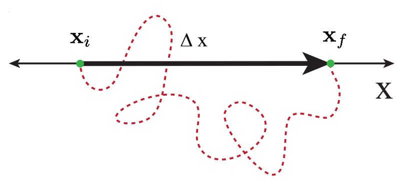
The particle model

This is our first real abstraction. Again, since we are trying to predict everything, we would like to figure out the rules that describe how any object would move. Take a train for example. If we asked a question like "when does the C train enter 59th street station?", a natural follow-up would be "well, do you mean the front of the train, or the middle of the train, or the end of the train? Each of these answers might be different by a few seconds.

How do we deal with this? By considering the train to be a 'point', we can neglect the actual length of the train and focus on what's more interesting: how the train moves.

The goal is to find the underlying physics that describes *all* trains. Once we do that, then we can improve our model by including information about the length of the individual train we are interested in.

2. Displacement Vector



To quantify the motion, we'll start by defining the **displacement vector**.

$$\Delta\mathbf{x} = \mathbf{x} - \mathbf{x}_0$$

In the case of our wandering bug, this would be the difference between the final position and the initial position.

This figure shows the displacement vector $\Delta\mathbf{x}$. This might be different than the distance traveled by the bug (shown in the dotted line).

Note on Notation!

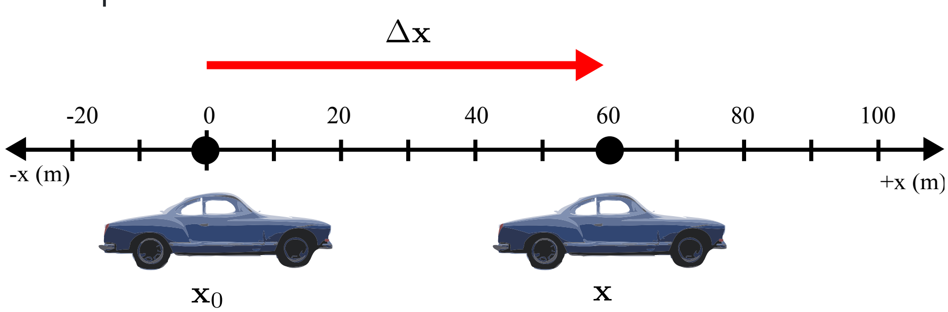
x_f is the same thing as x

x_i is the same thing as x_0

When describing motions, we usually have an initial position and a final position. We can call these x_i and x_f respectively, when we do our algebra.

Or another way of writing these quantities is to say our initial position is x_0 and our final position is just x. This is a slightly more general way of writing things.

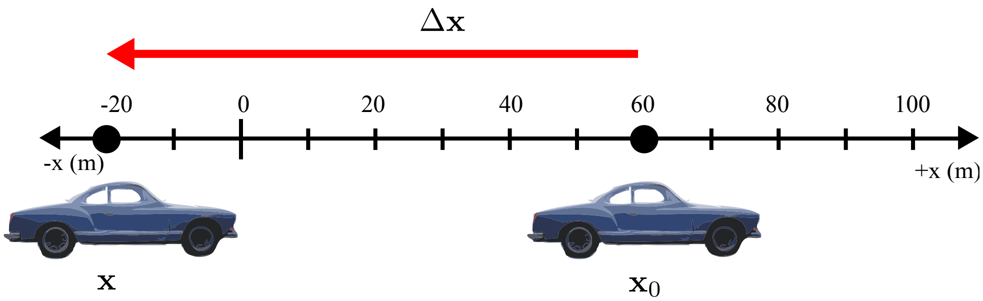
2.5 Displacement in 1-D



Here's a car that moves from $\mathbf{x}_0$ to $\mathbf{x}$ creating a displacement vector of:

$$\Delta\mathbf{x} = \mathbf{x} - \mathbf{x}_0 = 60 \text{ m} - 0 \text{ m} = 60 \text{ m}$$

The car then reverses to $x = -20$.



The leads to a displacement vector of $\Delta \mathbf{x} = -80\text{m}$.

About notation. Δx ("delta x") refers to the change in x. That is, difference between a final and initial value:

$$\Delta x = x - x_0$$

Or, in words, the final x position minus the original x position is equal to the change in x.

2.6 Distance Traveled

To get the *distance traveled*, we just neet to take the magnitude of the displacement during a certain motion.

$$|\Delta \mathbf{x}| = \text{Distance Traveled}$$

This equation will only be true if the displacement is always in the same direction. If however, the displacement vector were to change direction during a trip, the the distance traveled might not be equal to the total displacement. For example, if you walk 100 feet forward, then turn around and walk 50 backwards. You displacement from the initial to final position will only be 50 feet, but you will have walked a total of 150 feet.

3. Speed and Velocity

$$\boxed{\text{Average Speed} \equiv \frac{\text{Distance in a given time}}{\text{Elapsed time}}}$$

The 'elapsed time' is determined in the same way as the distance: $\Delta t = t - t_0$.

Again, t_0 is the starting time, and t is the final time.

Example Problem #1:

> Taking the A train between 59th and 125th takes about 8 minutes. The C, which is a local, takes 12 minutes (on a good day). Find the average speed for both of these trips.

3.7 ...with a direction

Calculating the average speed didn't tell us anything about the direction of travel. For this, we'll need average velocity.

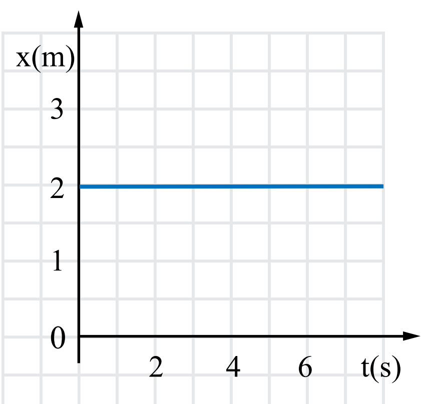
$$\boxed{\text{Average Velocity} \equiv \frac{\text{Displacement}}{\text{Elapsed time}}}$$

In mathematical terms:

$$\boxed{\bar{\mathbf{v}} \equiv \frac{\mathbf{x} - \mathbf{x}_0}{t - t_0} = \frac{\Delta \mathbf{x}}{\Delta t}}$$

(SI units of average velocity are m/s)
In one-dimension, velocity can either be in the positive or negative direction.

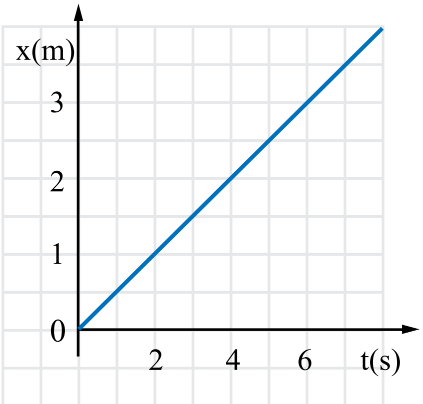
Quick Question 1



This is a graph showing the position of an object with respect to time.
Which choice best describes this motion?

a) The object is moving at 0.5 m/s in the +x direction.
b) The object is moving at 1.0 m/s in the +x direction.
c) The object is moving at 2.0 m/s in the +x direction.
d) The object is not moving at all.

Quick Question 2



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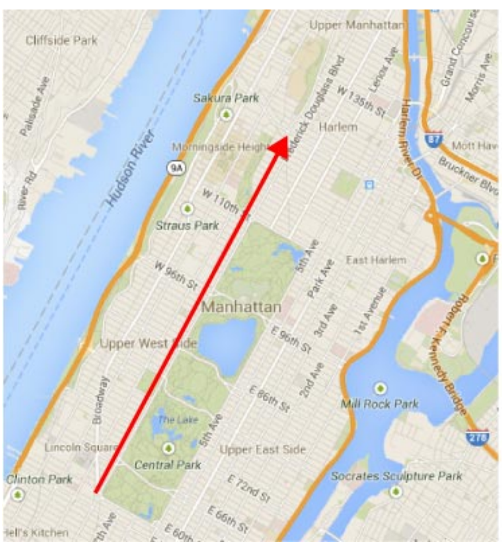
c) The object is moving at 2.0 m/s in the +x direction.
d) The object is not moving at all.

Thinking about the A train, it's clear that its speed and velocity stayed essentially constant between 59th and 125th ideally). However, the C train had to start and stop at 7 stations. To quantify, this difference in motion, we'll need to introduce the concept of *instantaneous velocity*.



If we imagine making many measurements of the velocity over the course of the travel, by reducing the Δx we are considering, then we can begin to see how we can more accurately assess the motion of the train.

The concept of instantaneous velocity involves considering an *infinitesimally small* section of the motion:

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{x}}{\Delta t} = \frac{dx}{dt}$$

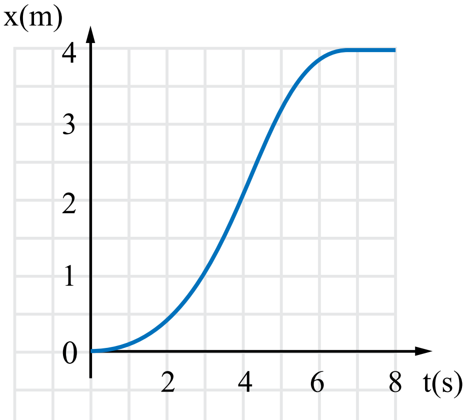
This will enable us to talk about the velocity at a particle's specific position or time rather than for an entire trip.

In general, this is what we'll mean when we say 'velocity' or 'speed'.

Science Sims

The Line Tangent to a Function ?

Quick Question 3



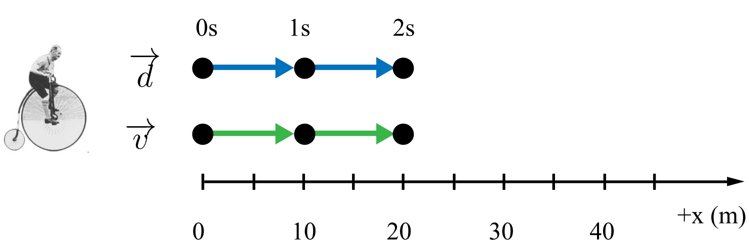
At which of the following times is the speed of this object the greatest?

a) t = 0
b) t = 2 s
c) t = 4 s
d) t = 8 s

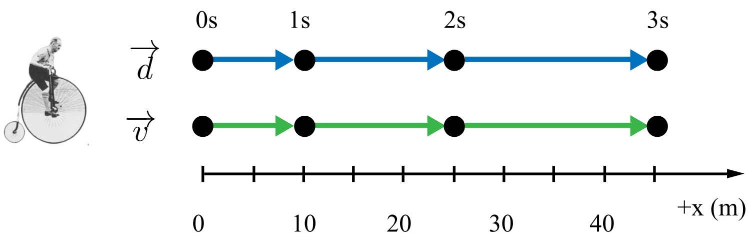
4. Change in velocity.

Naturally, in order to begin moving, an object must *change its velocity*.

Here's a graph of a bicyclist riding at a constant velocity. (In this case it's 10 m/s)



Now, here's a graph of the same bicyclist riding and changing his velocity during the motion



In the upper motion graph, notice how the length of the displacement vector $\vec{d}$ is the same at each interval in time. Meaning, that after 1 second has passed, the displacement is 10 m, after another second passes, another 10 meters displacement has occurred, making the total displacement equal to 20 m. This is motion at a **constant velocity**. This also apparent in the length of the velocity vectors at each point. They are always the same.

In the bottom graph, the displacement, and velocity vectors, change each time they are measured. This is representative of motion with **non-constant velocity**. The velocity is changing as time moves on.

4.8 Acceleration

This change in velocity we'll call **acceleration**, and we can define it in a very similar way to our definition of velocity:

$$\bar{\mathbf{a}} = \frac{\mathbf{v} - \mathbf{v}_0}{t - t_0} = \frac{\Delta \mathbf{v}}{\Delta t}$$

Again, in this case we're talking about *average* acceleration.

> Example Problem #2:
>
> At $t = 0$, the A train is at rest at 59th street. 5 seconds later, it's traveling north at 19 meters per second. What is the average acceleration during this time interval?

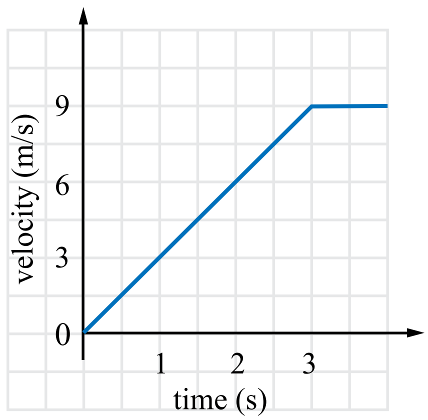
If we considered the same very small change in time, the infinitesimal change, then we could talk about instantaneous acceleration

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{dv}{dt}$$

The SI units of acceleration are meters per second per second, or ms^{-2}. That's probably a little bit of a weird unit, but, it makes sense to think about like this:

$$\frac{\left(\frac{m}{s}\right)}{s} \text{ or } \frac{vel}{s}$$

Quick Question 4



This is a graph showing the velocity of an object with respect to time.
Which choice best describes this motion?

a) The object is moving at the same velocity, which is 3 m/s.
b) The object starts at rest, and increases its velocity, for ever.
c) The object starts at rest, then increases its velocity for a while, then stops moving after 3 seconds.
d) The object starts at rest, then increases its velocity, then moves at the same speed after t = 3s.

4.9 Acceleration, the math.

To quantify to the acceleration of a moving body, say this car, we'll need to know its initial and final velocities

The car has a build in speedometer, so we can look at that to get the speed, and if we don't change direction, then the velocity will be always pointed in the same direction.

For this case of a car starting from rest, and then increasing velocity, the acceleration will be a positive quantity.

$$\bar{\mathbf{a}} = \frac{\mathbf{v} - \mathbf{v}_0}{t - t_0} = \frac{20\text{mph} - 0\text{mph}}{2\text{s} - 0\text{s}} = \frac{20\text{mph}}{2\text{s}}$$

$$\bar{\mathbf{a}} = \frac{9\text{m/s}}{2\text{s}} = +4.5\text{ms}^{-2}$$

4.10 Slowing down

What if we ask about a car slowing down. Now, our $\mathbf{v}_0 = +9m/s$ while $\mathbf{v} = 0$.

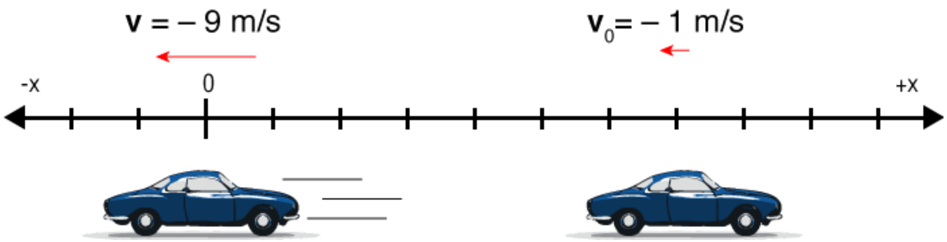
Now the math looks like this:

$$\bar{\mathbf{a}} = \frac{\mathbf{v} - \mathbf{v}_0}{t - t_0} = \frac{0\text{m/s} - 9\text{m/s}}{2\text{s} - 0\text{s}} = -\frac{9\text{m/s}}{2\text{s}} = -4.5\text{m/s}^2$$

We notice that the acceleration is *negative*.

4.11 Acceleration in the negative

What if the car starts accelerating in the negative direction?

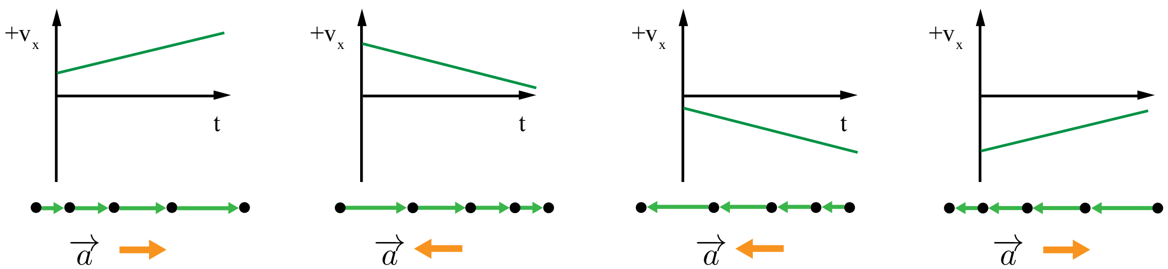


Now, even the speed is *increasing*, the velocity is getting *more negative*.

If we do the math, we'll see that the acceleration vector points in the negative direction.

4.12 Summary of acceleration signage.

When the signs of an object's velocity and acceleration are the same (in same direction), the object is speeding up
When the signs of an object's velocity and acceleration are opposite (in opposite directions), the object is slowing down and speed decreases



Quick Question 5

At one particular moment, a subway train is moving with a positive velocity and negative acceleration. Which of the following phrases best describes the motion of this train? Assume the front of the train is pointing in the positive x direction.

a) The train is moving forward as it slows down.
b) The train is moving in reverse as it slows down.
c) The train is moving faster as it moves forward.
d) The train is moving faster as it moves in reverse.
e) There is no way to determine whether the train is moving forward or in reverse.

Quick Question 6

At one particular moment, a subway train is moving with a negative velocity and positive acceleration. Which of the following phrases best describes the motion of this train? Assume the front of the train is pointing in the positive x direction.

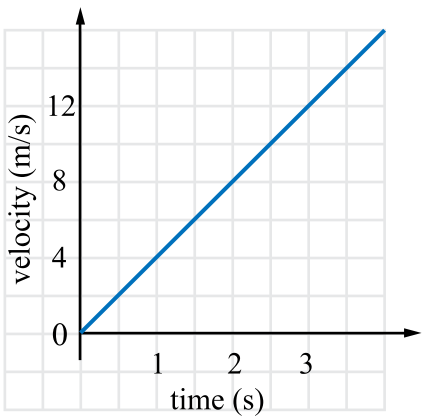
a) The train is moving forward as it slows down.
b) The train is moving in reverse as it slows down.
c) The train is moving faster as it moves forward.
d) The train is moving faster as it moves in reverse.
e) There is no way to determine whether the train is moving forward or in reverse.

Quick Question 7

A car is moving in the negative direction but slowing down. What is the sign of the car's acceleration?

a) Positive
b) Negative
c) Acceleration is equal to 0.

Quick Question 8



What is the average velocity of this object between 0 and 3 seconds?

a) 3 m/s
b) 4 m/s
c) 6 m/s
d) 12 m/s

5. Kinematic equations

1.

$$\bar{a} = a = \frac{v - v_0}{t} \quad \Rightarrow \quad v = v_0 + at$$

2.

$$\bar{v} = \frac{x - x_0}{t - t_0} \quad \Rightarrow \quad x - x_0 = \bar{v}t = \frac{1}{2}(v_0 + v)t$$

We can do a lot by rearranging these equations.

Putting v from (1) into (2) will give us:

3.

$$x - x_0 = v_0 t + \frac{1}{2}at^2$$

or, solving (1) for t, then inserting that into (2) will give us:

4.

$$v^2 = v_0^2 + 2a(x - x_0)$$

1. $v = v_0 + at$

2. $x = \bar{v}t = \frac{1}{2}(v_0 + v)t$

3. $x = x_0 + v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2ax$

$$\boxed{v = v_0 + at}$$

Here we have an equation for *velocity* which is changing due to an acceleration, a.

It tells us how fast something will be going (and the direction) if has been accelerated for a time, t.

- It can determine an object's velocity at any time t when we know its initial velocity and its acceleration
- Does not require *or give* any information about position
- Ex: "How fast was the car going after 10 seconds while accelerating from rest at 10 m/s^2"
- Ex: "How long did it take to reach 20 miles per hour"

$$\boxed{x = \bar{v}t = \frac{(v + v_0)t}{2}}$$

This equation will tell us the position of an object based on the initial and final velocities, and the time elapsed.

It does not require knowing, nor will it give you, the acceleration of the object.

- Ex: How far did the duck walk if it took 10 seconds to reach 50 miles per hour under constant acceleration.

$$\boxed{x = x_0 + v_0 t + \frac{at^2}{2}}$$

Gives position at time t in terms of initial velocity and acceleration

- Doesn't require or give final velocity.
- Ex: "How far up did the rocket go?"

$$\boxed{v^2 = v_0^2 + 2ax}$$

Gives velocity at time t in terms of acceleration and position

- Does not require or give any information about the time.
- Ex: "How fast was penny going when it reached the bottom of the well?"

5.13 Equations of Motion (1-D)

Things to be aware of:

1. They are *only* for situations where the acceleration is constant.
2. The way we have written them is really just for 1-D motion.

Equation	Missing Variable	Good for finding
$v = v_0 + at$	x	a,t,v
$x = \frac{(v+v_0)t}{2}$	a	x,t,v
$x = x_0 + v_0 t + \frac{at^2}{2}$	v	x,a,t
$v^2 = v_0^2 + 2ax$	t	a,x,v

6. Solving Problems

1. Diagram: draw a picture
2. Characters: Consider the problem a story. Who are the characters?
3. Find: clearly list symbolically what we're looking for.
4. Solve: state the basic idea behind solution, in a few words (physical principles used, etc.)
5. Assess: does answer make sense?

> Example Problem #3:
>
> A taxi is sitting at a red light. The light turns green and the taxi accelerates at 2.5 m/s^2 for 3 seconds. How far does it travel during this time?

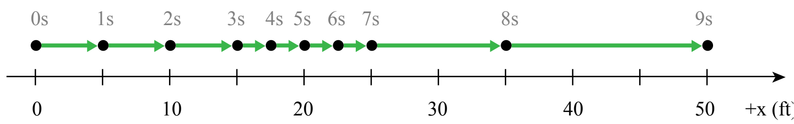
> Example Problem #4:
>
> A particle is at rest. What acceleration value should we give it so that it will be 2 meters away from its starting position after 0.4 seconds?

> Example Problem #5:
>
> A subway train accelerates starting at x = 200 m uniformly until it reaches x = 350 m, at a uniform acceleration value of 0.5 m/s^2.
>
> a. If it had an initial velocity of 0 m/s, what will the duration of this acceleration be?
> b. If it had an initial velocity of 8 m/s, what will the duration of this acceleration be?

> Example Problem #6:

> Traian Vuia, a Romanian Inventor, wanted to reach 17 m/s in order to take off in his flying machine. His plane could accelerate at $2 m/s^2$. The only runway he had access to was 80 meters long. Will he reach the necessary speed?

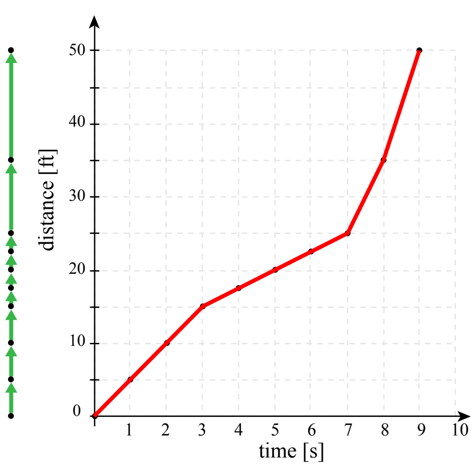
7. Plotting



Let's look at the motion of a honey badger.

After each second, we note where the honey badger is along the x axis.

t[s]	x[ft]
0	0.00
1	5.00
2	10.0
3	15.0
4	17.5
5	20.0
6	22.5
7	25.0
8	35.0
9	50.0



7.14 Plots -> Equations

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{x}}{\Delta t} = \frac{dx}{dt}$$

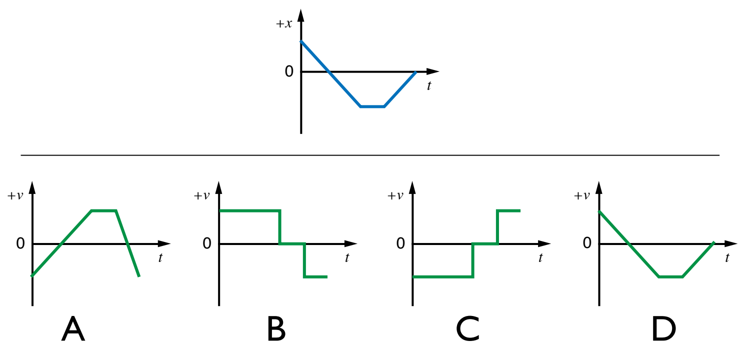
Velocity is the slope of position (w.r.t time)

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{dv}{dt}$$

Acceleration is the slope of velocity (w.r.t time)

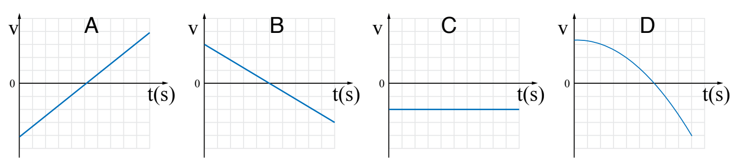
Quick Question 9

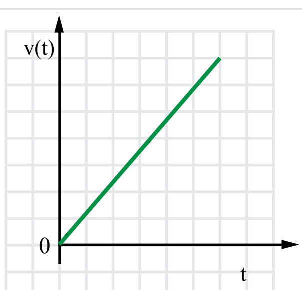
The first plot shows the position vs. time graph for an object in motion. Which of the velocity vs. time graphs shown would best correspond to this motion?

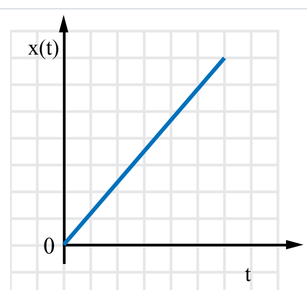


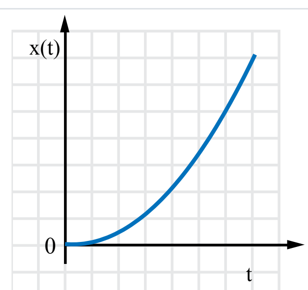
Quick Question 10

Which of the following velocity vs. time graphs represents an object with a negative constant acceleration?









$$v = v_0 + at$$

$$x = vt$$

$$x = v_0 t + \frac{at^2}{2}$$

velocity as a function of time: $v(t)$
Acceleration is constant

position as a function of time $x(t)$.
(vel. constant, accel = 0)

position as a function of time

Quick Question 11

Question 4: An object in motion

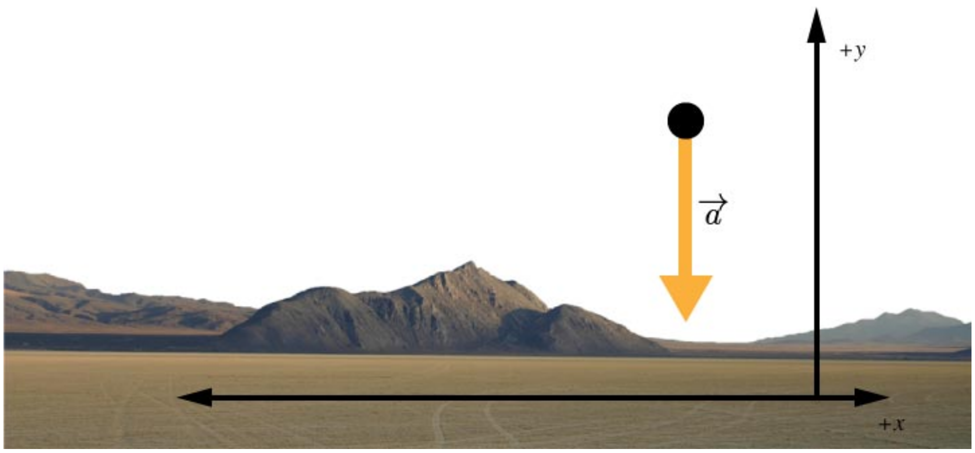
8. Free Fall

A freely falling object is any object moving freely under the influence of gravity alone.

Object could be:

1. Dropped = released from rest
2. Thrown downward
3. Thrown upward

It does not depend upon the initial motion of the object.



1. The acceleration of an object in free fall is directed downward (negative direction), regardless of the initial motion.
2. The magnitude of free fall acceleration is $\boxed{9.8\text{m/s}^2 = g}$.
3. We can neglect air resistance.
4. We'll choose our y axis to be positive upward.
5. Consider motion near Earth's surface for now.

Kinematic equation in the case of free fall:

1. $v = v_0 - gt$
2. $y = \bar{v}t = \frac{1}{2}(v_0 + v)t$
3. $y = y_0 + v_0 t - \frac{1}{2}gt^2$
4. $v^2 = v_0^2 - 2gy$

They are the same. We just replaced $x \rightarrow y$ and $a \rightarrow -g$.

Loading...
Ball Drop w/ Height Measure

?

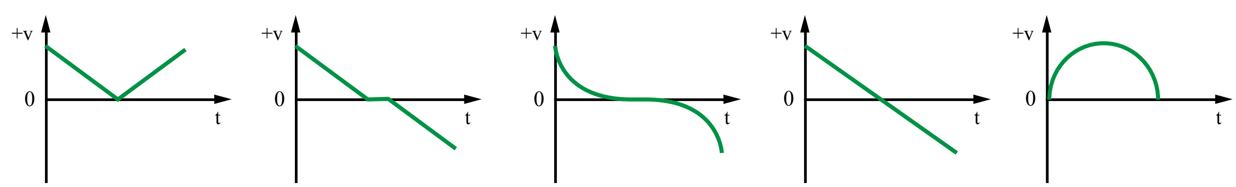
Example Problem #7:

An object is thrown upward at 20 m/s:

a. How long will it take to reach the top
b. How high is the top?
c. How long to reach the bottom?
d. How fast will it be going when it reaches the bottom?

Quick Question 12

An arrow is launched vertically upward. It moves straight up to a maximum height, then falls to the ground. Which graph best represents the vertical velocity of the arrow as a function of time? Ignore air resistance; the arrow is in free fall!.



A B C D E

Example Problem #8:

If an object is thrown upward from a height y_0 with a speed v_0, when will it hit the ground?

Example Problem #9:

8.15 Drop a wrench

A worker drops a wrench down the elevator shaft of a tall building.

> a. Where is the wrench 1.5 seconds later?
> b. How fast is the wrench falling at that time?

> Example Problem #10:
>
> A rock is thrown upward with a velocity of 49 m/s from a point 15 m above the ground.
>
> a. When does the rock reach its maximum height?
> b. What is the maximum height reached?
> c. When does the rock hit the ground?

8.16 How high was this?

> Example Problem #11:
>
> Draw position, velocity, and acceleration graphs as a functions of time, for an object that is let go from rest off the side of a cliff.