

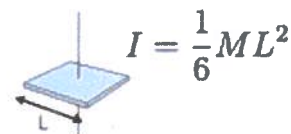
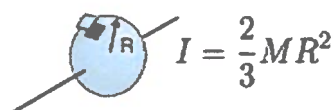
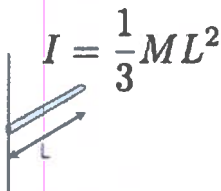
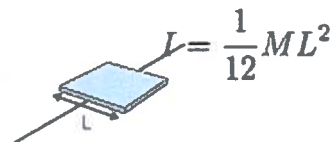
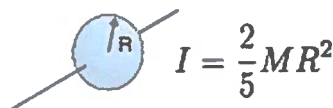
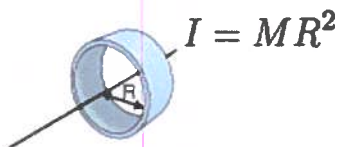
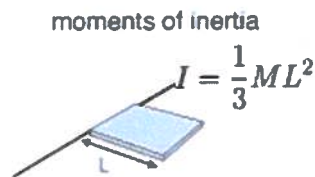
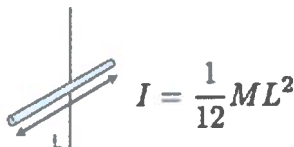
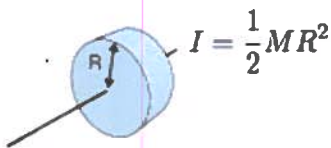
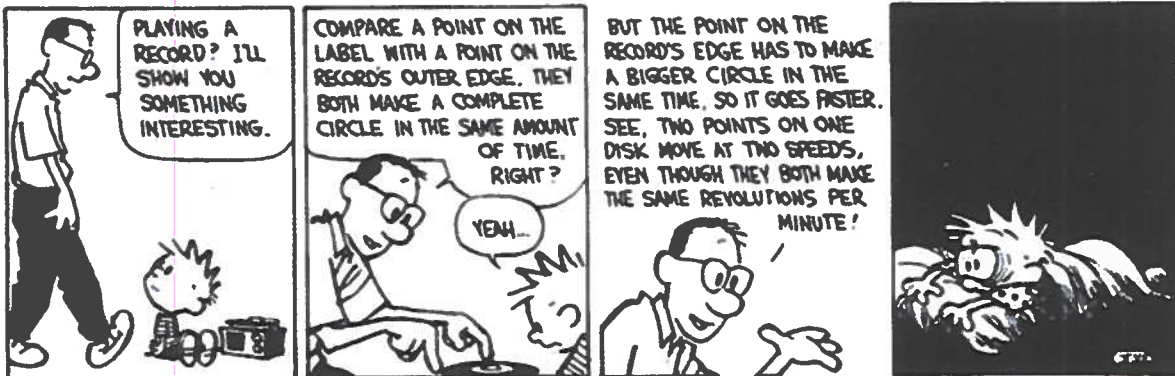
Instructions:

Short Answer problems: (3 pts/part) Show your work on the test for the short answer problems. You must show your work to get credit for the short answer problems. Each problem is worth the same amount. Include the SI units in your answers unless otherwise indicated. When doing the problems, the more steps you justify, the more likely you are to receive full credit, in other words, please show your work. Show *how you know*, not just *that you know*.

Multiple Choice: (2 pts each) Pick the best answer from the options available. You do not need to show any work for the multiple choice problems, however, you must fill in the circle for your final choice **or else no credit will be given**. Please do not lightly fill in several circles hoping that the vagueness of your selection can be used as a means of claiming points later (yes, people have done this). If your answers are not obviously indicated they may be marked incorrect.

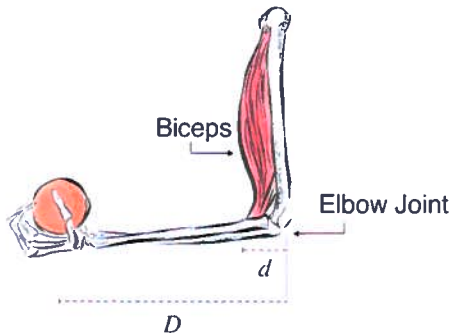


Graphical Exercise: (2 pts per graph) Sketch plots that show the relations between quantities on the two axes. Make your lines clear: if it's supposed to be a linear dependence, make the line clearly straight. If it's supposed to be quadratic, or something else non-linear, make the curves obvious.

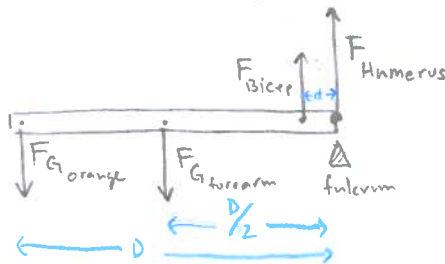


1. My arm is tired

Here is a skeletal diagram of an arm holding an orange. The arm is essentially a lever with a fulcrum at the elbow joint. The bicep connects to the forearm very close to the joint (length d) and provide the upward force needed to hold the arm out. The orange is located a distance D away from the joint. The total length of the forearm can be considered to be of length D as well.



a. Draw a simplified version of this situation as an (extended) free body diagram showing the various forces acting on the forearm. Make sure to include the mass of the forearm itself, in addition to the orange and the bicep forces. Write out the sums of torques acting and find an expression for the force needed to be exerted by the biceps in order to hold the orange stationary.



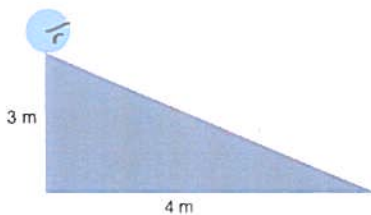
$$\Sigma \tau = F_{G0} D + F_{GFA} \frac{D}{2} - F_B d - F_{\text{Humerus}} (0) = 0$$

acts at fulcrum so no torque

solve for $F_B \Rightarrow F_B = \frac{D(F_{G0} + \frac{1}{2}F_{GFA})}{d}$

2. Rollin'

A solid sphere with a radius of 10 cm rolls the down ramp as shown.



a. What will its linear (m/s) velocity be when it reaches the bottom?

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad \text{Conservation of Energy}$$

$$\omega = \frac{v}{r} \quad \therefore \omega^2 = \frac{v^2}{r^2} \quad \therefore I_{\text{sphere}} = \frac{2}{5}Mr^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2} \cdot \frac{2}{5}Mr^2 \cdot \frac{v^2}{r^2} = \left(\frac{1}{2} + \frac{2}{10}\right)mv^2$$

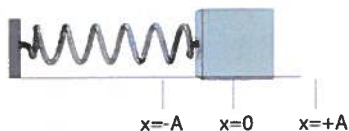
$$mgh = \frac{7}{10}mv^2 \Rightarrow v = \sqrt{\frac{10}{7}gh} = 6.48 \text{ m/s}$$

b. What will its angular velocity (rad/s) be when it reaches the bottom?

$$\omega = \frac{v}{r}$$

$$\omega = \frac{6.48}{0.1} = 64.8 \frac{\text{rad}}{\text{s}}$$

3. Spring Time



A horizontal spring/mass system is set up as shown. The spring constant is 25.8 N/m, the mass is 5.9 kg. The point A is 10 centimeters from the equilibrium position. The spring is extended until the mass reaches point A, then let go (at $t = 0$) and the system oscillates in Simple Harmonic Motion.

- a. How much work would be done in pulling the spring from equilibrium to the point A before letting go?

$$W = \Delta U = \frac{1}{2} k x^2 = \frac{1}{2} (25.8) (.1)^2 = 0.129 \text{ J}$$

- b. What is the speed of the mass when it passes through the point $x=0$?

$\checkmark 1$ speed at $x=0$ will be $v_{\max} = A\omega$ $\checkmark 2$ or
 $\omega = \sqrt{\frac{k}{m}} = 2.09 \frac{\text{rad}}{\text{s}}$ $\frac{1}{2} k x^2 = \frac{1}{2} m v^2$
 $\therefore A\omega = .209 \frac{\text{m}}{\text{s}}$ $0.129 = \frac{1}{2} m v^2$
 $v = \sqrt{\frac{2 \times 0.129}{m}} = 0.209 \frac{\text{m}}{\text{s}}$

← same result →

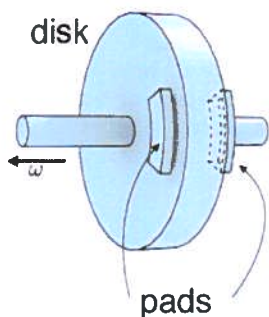
- c. What is the mass's kinetic energy after 1.5 seconds has passed since letting go?

$T = 2\pi \sqrt{\frac{m}{k}} = 3.00 \text{ seconds}$, so 1.5 seconds is exactly half the period, so the mass will be at $x = -A$, i.e. at the maximum displacement, so, $v = 0$, and $KE = 0$.
 -or-
 $v = -A \sin(\omega t)$
 $= -.1 \sin(2.09 \times 1.5) \approx 0.00 \text{ m/s}$

- d. How many oscillations will occur in 1 minute?

$$1 \text{ minute} = 60 \text{ seconds}$$

$$\frac{60 \text{ sec}}{3 \text{ sec/osc}} = 20 \text{ osc}$$



4. A disk brake works by squeezing two pads against a rotating disk. A large kinetic friction force is created between the pads and the disk, which reduces the angular velocity of the disk. Use the following parameters to answer the questions below:

Mass of disk: 2 kg
Radius of disk: 20 cm
Location of pad from edge: 4 cm

a. If the disc is rotating at 1000 rpm, and we wish to stop it in 3 seconds, calculate the torque needed to do this.

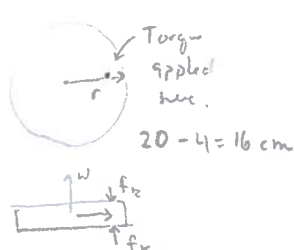
$$\tau = I\alpha \quad I_{\text{disc}} = \frac{1}{2}mr^2 = \frac{1}{2}(2\text{ kg})(.2)^2 = 0.04 \text{ kg m}^2$$

$$\frac{\Delta\omega}{\Delta t} = \alpha = \frac{1000 \text{ rpm}}{3 \text{ seconds}} = \frac{104.72 \text{ rad/s}}{3} = 34.9 \text{ rad/s}^2$$

convert

$$\text{thus } \tau = I\alpha = 0.04 \times 34.9 = 1.39 \text{ Nm}$$

b. If the coefficient of kinetic friction between the pads and the disk is $\mu_k = .8$, how much force should be applied to each brake pad to accomplish this?



$$f_k = F_N \mu_k$$

$$\tau = rF = r(2f_k)$$

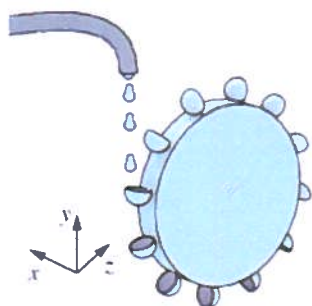
$$= .16(2 \times F_N \times 0.8) \Rightarrow F_N = 5.45 \text{ N}$$

solve for this

c. Estimate the amount of power required to stop this wheel in 3 seconds.

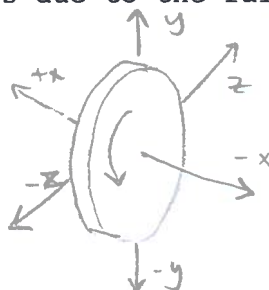
$$P = \frac{\text{CHANGE in Energy}}{\text{time}} = \frac{\Delta E}{3 \text{ sec}} = \frac{\frac{1}{2}I\omega^2}{3} = \frac{\frac{1}{2}(\frac{1}{2}mr^2)(104.72)^2}{3}$$

$$= 73.1 \frac{\text{J}}{\text{s}} = 73.1 \text{ Watts}$$



5. A water faucet drips water in the negative y direction as shown. What direction will the angular velocity of the wheel be, if it rotates due to the falling water?

- a. +x
- ☒ b. -x
- c. +y
- d. -y
- e. +z
- f. -z



6. Where would a person standing on the earth have the largest tangential velocity as the earth rotates in its daily motion?

- a. Close to the north pole
- b. In NYC
- ☒ c. Near the equator
- d. The tangential velocity would be the same everywhere.



equator has largest circumference

7. Where would a person standing on the earth have the largest angular velocity as the earth rotates in its daily motion?

- a. Close to the north pole
- b. In NYC
- c. Near the equator
- ☒ d. The angular velocity would be the same everywhere.

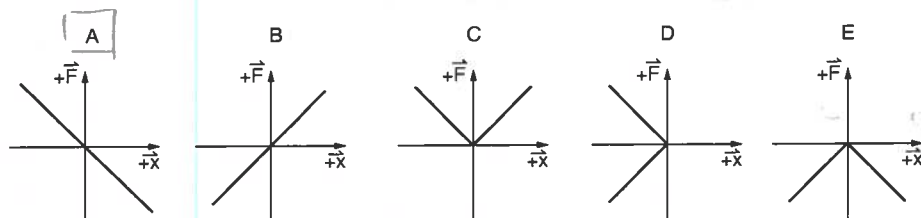
8. In an ideal spring mass system, the frequency of oscillation would change if you changed:

- i. The amplitude of oscillation.
- ii. The mass of the object that is oscillating
- iii. The spring constant

- a. i only
- b. ii only
- c. iii only
- ☒ d. ii and iii only
- e. i, ii, and iii

$$\omega = \sqrt{\frac{k}{m}}$$

9. Which of these plots would best represent Hooke's Law?



$$F = -kx$$



10. For an object undergoing simple harmonic motion:

- a. the displacement is greatest when the speed is greatest.
- b. the total energy oscillates at frequency $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$
- c. the acceleration is greatest when the speed is greatest.
- ☒ d. the acceleration is greatest when the displacement is greatest.
- e. the maximum potential energy is larger than the maximum kinetic energy.

11. A simple pendulum has a period T on Earth. If it were used on Planet X, where the acceleration due to gravity is 3 times what it is on Earth, its period would be:

- a. T
- b. $\sqrt{3}T$
- c. $T/3$
- d. $3T$
- e. $\frac{T}{\sqrt{3}}$

$$\left. \begin{aligned} T_{\text{pend}} &= 2\pi \sqrt{\frac{L}{g}} \\ T_x &= 2\pi \sqrt{\frac{L}{3g}} \end{aligned} \right\} \frac{T_x}{T} = \frac{2\pi \sqrt{\frac{L}{3g}}}{2\pi \sqrt{\frac{L}{g}}} = \frac{\sqrt{\frac{1}{3}}}{\sqrt{1}} = \frac{1}{\sqrt{3}}$$

$$T_x = \frac{T}{\sqrt{3}}$$

Multiple Choice Answers

Enter your answers in these boxes.

- 5. (A) (B) (C) (D) (E) (F)
- 6. (A) (B) (C) (D)
- 7. (A) (B) (C) (D)
- 8. (A) (B) (C) (D) (E)
- 9. (A) (B) (C) (D) (E)
- 10. (A) (B) (C) (D) (E)
- 11. (A) (B) (C) (D) (E)

12. Graphical Exercise

At time $t=0$, a mass on a stretched is released from a position of $+A$ meters, and begins to accelerate towards the equilibrium position. It then undergoes simple harmonic motion. Sketch 3 graphs for its position, velocity, and acceleration, for 2 complete oscillations, making sure the three graphs all use the same horizontal time scaling so that they are lined up as they should be.

motion of the mass

